Day 176 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves with constant acceleration. At t = 0 seconds P is at $(5\mathbf{i} - 5\mathbf{j})$ m with velocity $(-10\mathbf{i} + 6\mathbf{j})$ m s^{-1}. At t = 6, P has velocity $(8\mathbf{i} - 6\mathbf{j})$ m s^{-1}.
Find (a) the acceleration of P, (b) the time at which P moves parallel to $(\mathbf{i} - \mathbf{j})$, (c) the position vector of P at that time.

$a = \frac{v - u}{t} = \frac{(8\mathbf{i} - 6\mathbf{j}) - (-10\mathbf{i} + 6\mathbf{j})}{6} = (3\mathbf{i} - 2\mathbf{j})$

$v = (-10\mathbf{i} + 6\mathbf{j}) + t(3\mathbf{i} - 2\mathbf{j})$

Parallel to $(\mathbf{i} - \mathbf{j})$: $(3t - 10) \times -1 = (-2t + 6) \times 1$, giving t = 4

$r = (5\mathbf{i} - 5\mathbf{j}) + 4(-10\mathbf{i} + 6\mathbf{j}) + \frac{1}{2} \times 4^2 (3\mathbf{i} - 2\mathbf{j}) = (-11\mathbf{i} + 3\mathbf{j})$

Answer: (a) $(3\mathbf{i} - 2\mathbf{j})$ m s^{-2} (b) t = 4 (c) $(-11\mathbf{i} + 3\mathbf{j})$ m

2 Sectors: perimeter and area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius r cm and angle θ radians, where $\theta < 2\pi$.
Its perimeter is 42 cm and its area is 98 cm^2.
Find the two possible pairs of values of r and θ.

Arc = rθ, so $2r + r\theta = 42$ and $\frac{1}{2}r^2\theta = 98$

$r\theta = 42 - 2r$, so $\frac{1}{2}r(42 - 2r) = 98$

$r^2 - 21r + 98 = 0$, so $(r - 7)(r - 14) = 0$
r = 7: rθ = 28, θ = 4 r = 14: rθ = 14, θ = 1
Answer: r = 7, θ = 4 or r = 14, θ = 1

3 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(−3, 5) and B(−6, −4). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c - (-3))^2 + 25 = (c - (-6))^2 + 16$.
Expand and solve: −6c = 18, so c = −3.
Area = $\frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$

$= \frac{1}{2}|0 \times (-4) - (-3) \times 5| = \frac{15}{2}$.

Answer: C(−3, 0); area $\frac{15}{2}$

4 Triple angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE (triple angle))

Given that $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$, solve
$\cos 3\theta = 0$ for $0 < \theta < 180°$.

Substitute: $\cos\theta(4\cos^2\theta - 3 - 0) = 0$

$\cos\theta = 0$ or $\cos^2\theta = \frac{3}{4}$

$\cos\theta = 0$ gives 90°

$\cos^2\theta = \frac{3}{4}$ gives 30°, 150°

Answer: θ = 30°, 90°, 150°

5 Circle touching the axes (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle in the diagram touches both axes and passes through P. Find the equation of the circle, giving it in the form $x^2 + y^2 + ax + by + c = 0$.

Touches both axes in the first quadrant: centre (r, r), radius r.
P(6, 3) lies on the circle: $(6 - r)^2 + (3 - r)^2 = r^2$.
$r^2 - 18r + 45 = 0$, so r = 3 or r = 15.
The diagram shows P on the larger circle, so r = 15.
$(x - 15)^2 + (y - 15)^2 = 225$, then expand.
Answer: $x^2 + y^2 - 30x - 30y + 225 = 0$

6 Stationary points: exact coordinates (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the stationary point of
$y = x^2e^{-2x}$ for $x > 0$,
and determine its nature.

$dy/dx = xe^{-2x}(2 - 2x) = 0$ gives $x = 0$ or $x = 1$
$y = e^{-2}$ when $x = 1$
Second derivative at this point is negative, so it is a maximum
Answer: $(1, e^{-2})$, maximum

7 Velocity with a square-root term (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. At time t seconds, where $t \geq 0$, its velocity is v m s^{-1}, where
$v = 15\sqrt{t} - 3t$.
Find (a) the greatest speed of P, (b) the greatest displacement of P from its starting point.

$\frac{dv}{dt} = \frac{15}{2\sqrt{t}} - 3 = 0$ when $\sqrt{t} = \frac{5}{2}$, so $t = \frac{25}{4}$
$v = 15 \times \frac{5}{2} - 3 \times \frac{25}{4} = \frac{75}{4}$
$v = 0$ when $\sqrt{t}(15 - 3\sqrt{t}) = 0$: $t = 25$ ($t = 0$ is the start)
$s = \int v\,dt = \frac{2}{3} \times 15\, t^{\frac{3}{2}} - \frac{3}{2}t^2$, so at $t = 25$: $s = 1250 - \frac{1875}{2}$
Answer: (a) $\frac{75}{4}$ m s^{-1} (b) $\frac{625}{2}$ m

8 Conditional probability from given values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.2, P(B) = 0.15 and P(A | B) = 0.95.
(a) Find P(A ∩ B).
(b) Find P(A | B′).

$P(A \cap B) = P(A \mid B) \times P(B) = 0.95 \times 0.15 = 0.1425$
$P(A \cup B) = 0.2 + 0.15 - 0.1425 = 0.2075$
$P(B') = 0.85$
$P(A \cap B') = P(A) - P(A \cap B) = 0.0575$
$P(A \mid B') = \frac{0.0575}{0.85}$
Answer: (a) 0.1425 (b) $\frac{23}{340}$

9 Rational inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{2}{(x+3)} > \frac{3}{(x+1)}$.

Multiply by the positive quantity $(x + 3)^2(x + 1)^2$:
$2(x - (-3))(x - (-1))^2 > 3(x - (-1))(x - (-3))^2$.
Move to one side and factorise: $(x - (-3))(x - (-1))(-x - 7) > 0$.
Critical values: -3, -1, -7. Test the four regions.
Answer: $x < -7$ or $-3 < x < -1$

10 Conditional probability with a normal variable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A battery's life, in hours, is $N(69, 8^2)$.
Given that it exceeds 77, find the probability that it exceeds 86.

$\text{Required} = P(X > 86 \mid X > 77) = \frac{P(X > 86)}{P(X > 77)}$
$P(X > 86) = P(Z > 2.125) = 0.01679$
$P(X > 77) = P(Z > 1.000) = 0.1587$
$= \frac{0.01679}{0.1587} = 0.106$
Answer: 0.106