Day 175 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has sum to infinity 9 and second term -10. Find the possible values of the common ratio r and the first term a in each case.

$a = \frac{-10}{r}$. $S_\infty = \frac{a}{1-r} = \frac{-10}{r(1-r)} = 9$.

$9r^2 - 9r - 10 = 0$.

$(3r + 2)(3r - 5) = 0$: $r = -\frac{2}{3}$ or $r = \frac{5}{3}$.

Need $|r| < 1$: $-\frac{2}{3}$ valid, $\frac{5}{3}$ rejected.

Answer: $r = -\frac{2}{3}$, $a = 15$

2 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ax^n$ passes through (4, 64) and (16, 512).
Find the values of a and n.

$64 = a \times 4^n$ and $512 = a \times 16^n$

Divide: $4^n = 8$ so $n = \frac{3}{2}$

$a = 64 \div 4^{\frac{3}{2}} = 8$

Answer: $a = 8$, $n = \frac{3}{2}$

3 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^4 - x^3 - 15x^2 + 9x + 54 \leq 0$.

Use the factor theorem to find roots: -3, -2, 3.
$f(x) = (x + 3)(x + 2)(x - 3)^2$.
Sign of f in each region (left to right): +, −, +, +.
Take the regions where $f \leq 0$ (include roots if $\leq$ or $\geq$).
Answer: $-3 \leq x \leq -2$ or $x = 3$

4 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = t^2 - 3t$, $y = 2t^3 + t$.
Find the equation of the tangent at the point where t = 2,
in the form y = mx + c.

$dx/dt = 2t - 3$, $dy/dt = 6t^2 + 1$

At t = 2: $dy/dx = \frac{25}{1} = 25$; point $(-2, 18)$

$y - 18 = 25(x - -2)$
Answer: $y = 25x + 68$

5 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A die is said to show heads with probability 0.25. In 40 trials it does so 5 times.
Test at the 5% level whether the probability is different from 0.25.

H_0: $p = 0.25$, H_1: $p \neq 0.25$ (two-tailed). $X \sim B(40, 0.25)$
5 is below the mean 10, so use the lower tail: $P(X \leq 5) = 0.0433$
Each tail is compared with 2.5%: 0.0433 is greater than 2.5%
Do not reject H_0: there is insufficient evidence that the probability is different from 0.25
Answer: Do not reject H_0 (p-value 0.0433 against 2.5%)

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6 Rational inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{x+1}{-x+2} > 1$.

Rearrange: $\frac{x+1}{-x+2} - 1 > 0$.

Combine: $\frac{2x-1}{-x+2} > 0$.

Critical values: $x = \frac{1}{2}$ (zero of numerator), $x = 2$ (excluded, denominator zero).

Sign analysis of the three regions gives the solution.

Answer: $\mathbf{\frac{1}{2} < x < 2}$

7 Poisson hypothesis test (CIE)

Faults in a cable occur at random at a mean rate of 1.5 per day. After a modification, 3 faults are found in 1 day. Test at the 10% level whether the rate of faults has increased.

H_0: $\lambda = 1.5$ per day, H_1: $\lambda > 1.5$ per day

Over 1 day: $X \sim Po(1.5)$

$P(X \geq 3) = 1 - P(X \leq 2) = 0.191$

$0.191 > 10\%$, so do not reject H_0

There is insufficient evidence that the rate has increased

Answer: Do not reject H_0 (p-value 0.191)

8 Range and greatest height to speed and angle (AQA, Edexcel, OCR, MEI, Eduqas)

A golf ball is hit from a point O on horizontal ground. It lands 48 m from O, and reaches a maximum height of 30 m above the ground.

Find the angle of projection and the speed of projection, to 3 s.f. Take $g = 9.8$ m s^{-2}.

Vertical: $0 = u_y^2 - 2 \times 9.8 \times 30$, so $u_y = 24.25$

Time of flight $= \frac{2u_y}{9.8} = 4.949$ s, so $u_x = \frac{48}{4.949} = 9.699$

$\tan\theta = \frac{u_y}{u_x} = 2.500$, so $\theta = 68.2°$

Speed $= \sqrt{(u_x^2 + u_y^2)} = 26.1$

Answer: $\theta = 68.2°$, speed 26.1 m s^{-1}

9 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 + 3xy + 5y^2 = 495$.
Find the coordinates of the points on C where the tangent is parallel to the y-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x+3y}{3x+10y}$

Vertical tangent when $3x + 10y = 0$, so $x = -\frac{10y}{3}$

Substitute into the equation of C: $x^2 + (3xy) + (5y^2) = 495$

Solve for the remaining coordinate

Answer: $(-30, 9)$ and $(30, -9)$

10 Speed decaying exponentially (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. The velocity of P is v m s^{-1} and its acceleration is $-0.05v^2$ m s^{-2}. At t = 0, v = 20. Find (a) the time taken for the speed of P to halve, (b) the distance P travels in this time, to 3 s.f.

$\frac{dv}{dt} = -0.05v^2$: $\int \frac{dv}{v^2} = -0.05\int dt$, so $\frac{1}{v} = 0.05t + c$, and $\frac{1}{v} = 0.05t + \frac{1}{20}$

v = 10: $\frac{1}{v} = 0.1$, so $0.05t = 0.1 - 0.05$, giving t = 1

$s = \int \frac{1}{(0.05t + 1/20)}\, dt = \frac{1}{0.05} \ln(1 + 0.05 \times 20t)$

At t = 1: $s = \frac{1}{0.05} \ln 2$

Answer: (a) 1 s (b) 13.9 m