Day 174 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Compound change with logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A van costs £30000 and loses 15% of its value each year.
Find the least whole number of years after which its value is below £4000.

$30000 \times 0.85^n < 4000$, so $0.85^n < \frac{2}{15}$

$n > \frac{\ln \frac{2}{15}}{\ln 0.85} = 12.40$ (ln 0.85 is negative, so the inequality reverses)

$n = 13$

Answer: 13

2 Stone thrown from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 5 m s^{-1} from the top of a cliff. The top of the cliff is 29.1 m above the sea. Find (a) the time taken for the stone to reach the sea, (b) the total distance travelled by the stone, to 3 s.f. Take g = 9.8 m s^{-2}.

Take upwards as positive: $-29.1 = 5t - 4.9t^2$, so $4.9t^2 - 5t - 29.1 = 0$

$t = \frac{5 + \sqrt{(5^2 + 19.6 \times 29.1)}}{9.8} = \frac{5 + 24.4}{9.8} = 3$

Height risen above the cliff top = $\frac{5^2}{(2 \times 9.8)} = 1.28$ m

Total distance = $2 \times 1.28 + 29.1$

Answer: (a) 3 s (b) 31.7 m

3 Exponential growth and decay models (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A radioactive sample has mass $m = m_0 e^{-kt}$ grams, t in days.

Initially m = 100 and after 5 days $m = \frac{200}{3}$.

Find k, and the time for the mass to fall to 10 grams, to 3 s.f.

$m_0 = 100$. $100\, e^{-5k} = \frac{200}{3}$ so $e^{-5k} = \frac{2}{3}$

$k = \frac{1}{5} \ln \frac{3}{2} = 0.0811$

$100\, e^{-kt} = 10$ so $e^{kt} = 10$, $t = \frac{1}{k} \ln 10$

t = 28.4 days

Answer: k = 0.0811, t = 28.4 days

4 Least number of trials (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each battery is faulty with probability 0.05, independently.
Find the least number n of batterys that must be tested so that the probability of finding at least one faulty is greater than 0.995.

P(at least one faulty) = $1 - 0.95^n > 0.995$

$0.95^n < 0.005$

$n \ln 0.95 < \ln 0.005$

Dividing by ln 0.95 (negative) reverses the inequality: n > 103.3

n = 104

Answer: n = 104

5 Indices: quadratic in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2^{2x+2} - 9 \times 2^x + 2 = 0$.

Let $y = 2^x$. Then $4y^2 - 9y + 2 = 0$.

$4(y - \frac{1}{4})(y - 2) = 0$, so $y = \frac{1}{4}$ or $y = 2$.

$2^x = \frac{1}{4}$ gives $x = -2$; $2^x = 2$ gives $x = 1$.

Answer: x = −2 or x = 1

6 Rough slope: up then back down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 10 m s^{-1} up a line of greatest slope of a rough plane inclined at α to the horizontal, where $\sin\alpha = \frac{4}{5}$ and $\mu = 0.4$.

Find (a) the distance it travels up the plane, to 3 s.f., (b) its speed on returning to its starting point. Take g = 9.8 m s^{-2}.

$\cos\alpha = \frac{3}{5}$. Up the plane: $R = mg\cos\alpha$, so $a_{up} = g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{4}{5} + 0.4 \times \frac{3}{5}) = 10.192$

$0 = 10^2 - 2 \times 10.192 \times d$, so $d = \frac{100}{20.384} = 4.906$

Down the plane: $a_{down} = g(\sin\alpha - \mu\cos\alpha) = 9.8(\frac{4}{5} - 0.4 \times \frac{3}{5}) = 5.488$

$v^2 = 2 \times 5.488 \times 4.9058 = 53.85$, so $v = 7.34$

Answer: (a) 4.91 m (b) 7.34 m s^{-1}

7 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sum_{r=1}^{n}(4r - 3) = 1540$, find n.

$\sum_{r=1}^{n}(4r - 3) = 4 \times \frac{n(n+1)}{2} - 3n.$

$\frac{4\,n(n+1)}{2} - 3n = 1540$, so $4n^2 - 2n - 3080 = 0$.

Solving (n > 0): n = 28.

Answer: n = 28

8 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^3\sqrt{252 - x}$, for $0 < x < 252$, has one stationary point. Find its exact coordinates and determine its nature.

$\frac{dy}{dx} = \frac{x^2(1512 - 7x)}{2\sqrt{252 - x}}$

$\frac{dy}{dx} = 0$: $1512 - 7x = 0$, so $x = 216$

$y = 216^3 \times 6$

Gradient changes from positive to negative: a maximum

Answer: (216, 60466176), maximum

9 Surds: rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Show that $\frac{3+\sqrt{11}}{3-\sqrt{11}} - \frac{3-\sqrt{11}}{3+\sqrt{11}}$

can be written in the form $k\sqrt{11}$, and find k.

Common denominator $(3 - \sqrt{11})(3 + \sqrt{11}) = 9 - 11 = -2$.

Numerator: $(3 + \sqrt{11})^2 - (3 - \sqrt{11})^2$

$= (9 + 11 + 23\sqrt{11}) - (9 + 11 - 23\sqrt{11}) = 12\sqrt{11}$.

So the expression $= \frac{12\sqrt{11}}{-2}$.

Answer: k = −6

10 Poisson hypothesis test (CIE)

Faults in a cable occur at random at a mean rate of 2.5 per hour. After a modification, 12 faults are found in 2 hours. Test at the 10% level whether the rate of faults has increased.

H_0: $\lambda = 2.5$ per hour, H_1: $\lambda > 2.5$ per hour

Over 2 hours: $X \sim Po(5)$

$P(X \geq 12) = 1 - P(X \leq 11) = 0.00545$

$0.00545 < 10\%$, so reject H_0

There is sufficient evidence that the rate has increased

Answer: Reject H_0 (p-value 0.00545)