Day 169 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Logarithmic equations with ln (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2 \ln x = \ln(x + 2) + \ln 1$,
stating clearly why one root is rejected.

$\ln x^2 = \ln(1(x + 2))$, so $x^2 = 1x + 2$
$x^2 - 1x - 2 = 0$, $(x - 2)(x + 1) = 0$
$x = -1$ is rejected since $\ln x$ is undefined for $x < 0$
Answer: $x = 2$

2 Acceleration given, find when P returns to O (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time $t = 0$, P passes through the fixed point O with velocity -48 m s^{-1}. At time t seconds, P has acceleration $(-3t + 22)$ m s^{-2}.
Find (a) the times at which P is at O again, (b) the greatest distance of P from O before it first returns to O.

$v = \int a \, dt = -1.5t^2 + 22t - 48$ ($v = -48$ when $t = 0$)
$s = \int v \, dt = -0.5t^3 + 11t^2 - 48t$ ($s = 0$ when $t = 0$)
$s = -0.5t(t - 6)(t - 16)$, so P is at O when $t = 6$ and $t = 16$
Greatest distance when $v = 0$: $-1.5t^2 + 22t - 48 = 0$, so $t = \frac{8}{3}$ (the smaller root)
$|s| = 0.5 \times \frac{8}{3} \times \frac{10}{3} \times \frac{40}{3}$
Answer: (a) $t = 6$, $t = 16$ (b) $\frac{1600}{27}$ (= 59.3) m

3 Integrals of the form f'(x)/f(x) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$$\int_0^3 \frac{x}{x^2 + 1} \, dx$$

Numerator is $\frac{1}{2}$ × derivative of the denominator
Integral = $[\frac{1}{2}\ln(x^2 + 1)]$
$= \frac{1}{2}(\ln 10 - \ln 1)$
Answer: $\frac{1}{2}\ln 2 + \frac{1}{2}\ln 5$

4 Cubic graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the cubic $y = f(x)$, which touches the x-axis once. Find $f(x)$, giving your answer in expanded form.

Crosses at $x = -1$ and touches at $x = -2$, so $f(x) = k(x + 1)(x + 2)^2$.
At (0, 4): $4 = k(1)(2)^2 = 4k$, so $k = 1$.
$f(x) = (x + 1)(x + 2)^2$, then expand.
Answer: $f(x) = x^3 + 5x^2 + 8x + 4$

5 Symmetric limits of a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Scores are N(58, 9^2). The highest 15% and the lowest 15% of scores are discarded.
Find the range of scores that remain, correct to 3 significant figures.

The remaining middle 70% lies between $z = \pm 1.0364$
Lower limit = $58 - 1.0364 \times 3 = 54.89$
Upper limit = $58 + 1.0364 \times 3 = 61.11$
Answer: 54.9 to 61.1

6 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = y^3 + y^2 - 2y$.
Find dy/dx and d^2y/dx^2 when $y = 2$.

$dx/dy = 3y^2 + 2y - 2 = 14$ when $y = 2$, so $dy/dx = \frac{1}{dx/dy} = \frac{1}{14}$

$d^2x/dy^2 = 14$ when $y = 2$

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{14}{14^3}$

Answer: $dy/dx = \frac{1}{14}$, $d^2y/dx^2 = -\frac{1}{196}$

7 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation $y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes $x = -2$ and $y = 3$, so $p = 2$ and $q = 3$.

$y = \frac{a}{x + 2} + 3$.

P$(-4, 6)$: $6 = \frac{a}{-2} + 3$, so $a = -6$.

Answer: $a = -6$, $p = 2$, $q = 3$

8 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 473 inclusive that are divisible by 3 or by 7, but not by both.

Multiples of 3: 157 terms (3 to 471): sum = 37209.
Multiples of 7: 67 terms: sum = 15946.
Multiples of both, i.e. of 21: 22 terms: sum = 5313.
Required sum = $37209 + 15946 - 2 \times 5313$.
Answer: 42529

9 Speed decaying exponentially (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. The velocity of P is v m s^{-1} and its acceleration is $-0.05v^2$ m s^{-2}. At $t = 0$, $v = 4$. Find (a) the time taken for the speed of P to halve, (b) the distance P travels in this time, to 3 s.f.

$\frac{dv}{dt} = -0.05v^2$: $\int \frac{dv}{v^2} = -0.05\int dt$, so $\frac{1}{v} = 0.05t + c$, and $\frac{1}{v} = 0.05t + \frac{1}{4}$

$v = 2$: $\frac{1}{v} = 0.5$, so $0.05t = 0.5 - 0.25$, giving $t = 5$

$s = \int \frac{1}{(0.05t + 1/4)}\, dt = \frac{1}{0.05} \ln(1 + 0.05 \times 4t)$

At $t = 5$: $s = \frac{1}{0.05} \ln 2$

Answer: (a) 5 s (b) 13.9 m

10 Normal: find mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The scores in an aptitude test are normally distributed. 15% are less than 49 and 40% are greater than 66. Find the mean and standard deviation, each correct to 3 significant figures.

Let $X \sim N(\mu, \sigma^2)$. P$(X < 49) = 0.15$ gives $\frac{49 - \mu}{\sigma} = -1.0364$

P$(X > 66) = 0.40$ gives $\frac{66 - \mu}{\sigma} = 0.2533$

Subtract: $\frac{17}{\sigma} = 0.2533 - (-1.0364) = 1.2898$

$\sigma = 13.18$
$\mu = 49 - (-1.0364)(13.18) = 62.66$
Answer: $\mu = 62.7$, $\sigma = 13.2$