Day 163 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 31 and upper quartile 51. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.

(a) Find the range of values that are not outliers.

(b) 10 observations have mean 35. The largest is 85, an outlier, and it is removed. Find the new mean.

IQR = 51 − 31 = 20, so 1.5 × IQR = 30

Lower limit = 31 − 30 = 1, upper limit = 51 + 30 = 81

$\Sigma x = 10 \times 35 = 350$, so without 85: $\Sigma x = 265$

New mean = $\frac{265}{9}$

Answer: (a) 1 to 81 (b) 29.44

2 Cubic with only one real root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 2x^2 - 2x - 24$.

Show that $(x - 4)$ is a factor of $f(x)$, and that the equation $f(x) = 0$ has exactly one real root.

$f(4) = 64 - 32 - 8 - 24 = 0$, so $(x - 4)$ is a factor.

$f(x) = (x - 4)(x^2 + 2x + 6)$.

For $x^2 + 2x + 6 = 0$: $b^2 - 4ac = 4 - 24 = -20 < 0$.

No real roots from the quadratic, so the only real root is $x = 4$.

Answer: Proved; the only real root is x = 4

3 Sectors: perimeter and area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius r cm and angle θ radians, where $\theta < 2\pi$.

Its perimeter is 46 cm and its area is 126 cm^2.

Find the two possible pairs of values of r and θ.

Arc = $r\theta$, so $2r + r\theta = 46$ and $\frac{1}{2}r^2\theta = 126$

$r\theta = 46 - 2r$, so $\frac{1}{2}r(46 - 2r) = 126$

$r^2 - 23r + 126 = 0$, so $(r - 9)(r - 14) = 0$

$r = 9$: $r\theta = 28$, $\theta = \frac{28}{9}$ $r = 14$: $r\theta = 18$, $\theta = \frac{9}{7}$

Answer: r = 9, $\theta = \frac{28}{9}$ or r = 14, $\theta = \frac{9}{7}$

4 Stratified sample then probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A school has 60 students in Year 10, 60 in Year 11 and 120 in the sixth form.

A stratified sample of 60 students is taken.

(a) Find the number sampled from each group.

(b) Two students are chosen at random from the sample. Find the probability they are from the same group.

Sample sizes: $60 \times \frac{60}{240}$, $60 \times \frac{60}{240}$, $60 \times \frac{120}{240} = 15, 15, 30$

P(same) = 15 × 14 + 15 × 14 + 30 × 29/60 × 59

$= \frac{1290}{3540} = 0.364$

Answer: (a) 15, 15, 30 (b) 0.364

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5 Partial fractions (repeated factor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-14x^2 + 32x - 48}{(x + 4)(2x - 2)^2}$ in the form $\frac{A}{(x + 4)} + \frac{B}{(2x - 2)} + \frac{C}{(2x - 2)^2}$.

$-14x^2 + 32x - 48 \equiv A(2x - 2)^2 + B(x + 4)(2x - 2) + C(x + 4)$.

Put $x = -4$: $-400 = A(-10)^2$, so $A = -4$.

Put $x = 1$: $-30 = C(5)$, so $C = -6$.

Compare x^2: $-14 = 4A + 2B$, so $B = 1$.

Answer: $A = -4$, $B = 1$, $C = -6$

6 Non-uniform rod: centre of mass and tilting (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB of length 8 m and mass 8 kg rests on supports at C and D, where AC = 2 m and AD = 5 m, with R_C = 1.5 R_D.

Find (a) the distance of the centre of mass from A. A particle of mass m kg at B puts the rod on the point of tilting about D. (b) Find m.

R_C = 1.5 R_D. Moments about A: R_C × 2 + R_D × 5 = 8 g × AG

R_D(1.5 × 2 + 5) = 8 g × AG with R_D(1 + 1.5) = 8 g, so AG = $\frac{1.5 \times 2 + 5}{2.5} = 3.2$

Tilting about D: R_C = 0. Moments about D: m g $(8 - 5) = 8$ g $(5 - 3.2)$

m = $\frac{8 \times 1.8}{3} = 4.8$

Answer: (a) 3.2 m (b) 4.8

7 Stationary points of trigonometric curves (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the stationary points of
$y = 5x + 10 \cos x$ for $0 \leq x \leq \pi$,
and determine their nature.

$dy/dx = 5 - 10 \sin x = 0$, so $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

$d^2y/dx^2 = -10 \cos x$

Its sign at each point decides maximum or minimum

Answer: $\left(\frac{\pi}{6}, \frac{5}{6}\pi + 5\sqrt{3}\right)$ maximum; $\left(\frac{5\pi}{6}, \frac{25}{6}\pi - 5\sqrt{3}\right)$ minimum

8 Compound interest and savings (GP) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is dropped from a height of 4 m. After each bounce it rises to $\frac{1}{2}$ of its previous height.

(a) Find the total distance it travels before it stops.

(b) After how many bounces does it rise less than 0.01 m?

(a) Down H, then up and down each time: $H + 2Hr + 2Hr^2 + \ldots$

$= H + \frac{2Hr}{1 - r} = 4 + \frac{2 \times 4 \times \frac{1}{2}}{1 - \frac{1}{2}} = 12.$

(b) Rebound height after n bounces $= 4(\frac{1}{2})^n < 0.01$.

$n > \frac{\log(0.0025)}{\log(0.5)} = 8.64$, so $n = 9$.

Answer: (a) 12 m (b) 9 bounces

9 Just clearing a wall at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 45° to the horizontal. The ball just passes over the top of a vertical wall of height 27.2 m, at a horizontal distance of 40 m from O.

Find (a) the speed of projection, (b) the greatest height of the ball above the ground, to 3 s.f. Take g = 9.8 m s^{-2}.

At 45°: $x = \frac{u\,t}{\sqrt{2}}$ and $y = \frac{u\,t}{\sqrt{2}} - 4.9t^2$, so $y = x - \frac{9.8x^2}{u^2}$

$27.2 = 40 - \frac{9.8 \times 40^2}{u^2}$, so $u^2 = \frac{9.8 \times 1600}{12.8} = 1225$

u = 35

Greatest height = $\frac{(u \sin 45°)^2}{(2 \times 9.8)} = \frac{1225}{39.2}$

Answer: (a) 35 m s^{-1} (b) 31.2 m

10 Lines in 3D: intersection and angle (CIE (9709); Further Maths on other boards)

Line l_1: $\mathbf{r} = \begin{pmatrix}7\\3\\3\end{pmatrix} + s\begin{pmatrix}2\\2\\-2\end{pmatrix}$,

line l_2: $\mathbf{r} = \begin{pmatrix}2\\1\\7\end{pmatrix} + t\begin{pmatrix}3\\0\\-2\end{pmatrix}$.

Show whether the lines meet, giving the point of intersection if they do, and find the acute angle between them to 1 d.p.

Equate components: 7 + 2s = 2 + 3t; 3 + 2s = 1; 3 − 2s = 7 − 2t

Solving the first two: s = −1, t = 1; the third equation is satisfied

Intersection: (5, 1, 5)

$\cos\varphi = \frac{|10|}{\sqrt{12}\sqrt{13}} = 0.8006$, $\varphi = 36.8°$

Answer: Lines meet at (5, 1, 5); angle 36.8°