Day 162 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{3x + 7} = x + 1$.

Square both sides: $3x + 7 = (x + 1)^2$.

$x^2 - x - 6 = 0$, so $x = -2$ or $x = 3$.

Check in $\sqrt{3x + 7} = x + 1$: $x = 3$ gives $4 \geq 0$ (valid).

$x = -2$ gives right-hand side $-1 < 0$, so it is rejected.

Answer: $x = 3$

2 Poisson approximation to a binomial (CIE)

$X \sim B(400, 0.01)$.

(a) Explain why a Poisson approximation is suitable and state its distribution.

(b) Use it to estimate $P(X > 2)$, to 4 s.f.

(c) Find the exact value of $P(X > 2)$, to 4 s.f.

$n = 400$ is large and $p = 0.01$ is small, with $np = 4$

X is approximately Po(4)

$P(X > 2) \approx 1 - P(Y \leq 2) = 0.7619$

Exact: $1 - P(X \leq 2)$ with $X \sim B(400, 0.01) = 0.7634$

Answer: (a) Po(4) (b) 0.7619 (c) 0.7634

3 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sin(\theta + 40°) = -\frac{\sqrt{3}}{2}$

for $-90° \leq \theta \leq 270°$.

Let $u = \theta + 40°$; then $\sin u = -\frac{\sqrt{3}}{2}$

Base solutions: $u = 240°, -60°$ (+ multiples of 360°)

$\theta = (u - 40°) \div 1$, keeping those in range

Answer: $\theta = 200°, 260°$

4 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows $y = f(x)$, made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of

$y = 3f(2x - 8) - 4$.

Inside f: replace x by $2x - 8$, so x-coordinates become $(x + 8) \div 2$.

Outside: y-coordinates become $3 \times y - 4$.

A: $(1, -1)$ B: $(3, 11)$

C: $(6, -1)$ D: $(7, -16)$

Answer: A′(1, −1), B′(3, 11), C′(6, −1), D′(7, −16)

5 Distance travelled with a change of direction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time t seconds its velocity is v m s^{-1}, where

$v = t^2 - 5t + 6$ for $0 \leq t \leq 5$

Find (a) the times at which P is at instantaneous rest, (b) the total distance travelled by P in the first 5 seconds.

$v = (t - 2)(t - 3)$, so $v = 0$ when $t = 2$ and $t = 3$

$v > 0$ for $0 < t < 2$, $v < 0$ for $2 < t < 3$, $v > 0$ for $t > 3$: direction changes at each

$s = \int v\,dt = \frac{1}{3}t^3 - 2.5t^2 + 6t$

Distances: 0 to 2: $\frac{14}{3}$; 2 to 3: $\frac{1}{6}$; 3 to 5: $\frac{14}{3}$

Total $= \frac{14}{3} + \frac{1}{6} + \frac{14}{3}$

Answer: (a) $t = 2$, $t = 3$ (b) $\frac{19}{2}$ m

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6 Stationary points: exact coordinates (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the stationary points of

$y = \frac{x}{x^2 + 16}$,

and determine the nature of each.

$dy/dx = \frac{(x^2 + 16) - 2x^2}{(x^2 + 16)^2} = \frac{16 - x^2}{(x^2 + 16)^2} = 0$ gives $x = \pm 4$

$y = \pm\frac{1}{8}$ at $x = \pm 4$

d^2y/dx^2 is negative at $x = 4$ and positive at $x = -4$

Answer: $(4, \frac{1}{8})$ maximum; $(-4, -\frac{1}{8})$ minimum

7 Overtaking with a delayed start (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O travelling at a constant speed of 12.8 m s^{-1}. 4 s later, car B leaves O from rest with constant acceleration 2 m s^{-2}, travelling in the same direction.
B overtakes A at the point X. Find (a) the time B takes to reach X, (b) the distance OX, (c) the speed of B at X.

Let B travel for t s. A: distance = 12.8(t + 4); B: distance = ½ × 2 × t^2

Equate: $2t^2 - 25.6t - 102.4 = 0$, so $(t - 16)(2t + 6.4) = 0$

t = 16 (the other root is negative)

OX = 2 × 16^2 / 2 = 256 m, and speed = 2 × 16 = 32

Answer: (a) 16 s (b) 256 m (c) 32 m s^{-1}

8 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 + xy + 4y^2 = 60$.
Find the coordinates of the points on C where the tangent is parallel to the y-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + y}{x + 8y}$

Vertical tangent when $x + 8y = 0$, so $x = -\frac{8y}{1}$

Substitute into the equation of C: $x^2 + (xy) + (4y^2) = 60$

Solve for the remaining coordinate

Answer: $(-8, 1)$ and $(8, -1)$

9 Binomial expansion (fractional or negative powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the binomial expansion of $(9 + 6x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3.
State the values of x for which it is valid.

$(9 + 6x)^{\frac{3}{2}} = 9^{\frac{3}{2}}(1 + \frac{2}{3}x)^{\frac{3}{2}} = 27(1 + u)^{\frac{3}{2}}$, $u = \frac{2}{3}x$.

$(1 + u)^{\frac{3}{2}} = 1 + \frac{3}{2}u + \frac{3}{8}u^2 + (-\frac{1}{16})u^3 + ...$

Substitute u and multiply by 27.

Valid when $|u| < 1$, i.e. $|x| < \frac{3}{2}$.

Answer: $27 + 27x + \frac{9}{2}x^2 - \frac{1}{2}x^3 + ...$; $|x| < \frac{3}{2}$

10 Conditional probability with a normal variable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A student's time on a task, in minutes, is N(70, 4^2).
Given that it exceeds 74, find the probability that it exceeds 77.

Required = $P(X > 77 \mid X > 74) = \frac{P(X > 77)}{P(X > 74)}$

$P(X > 77) = P(Z > 1.750) = 0.04006$

$P(X > 74) = P(Z > 1.000) = 0.1587$

$= \frac{0.04006}{0.1587} = 0.252$

Answer: 0.252