Day 161 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{4x + 9} = 2x - 3$.

Square both sides: $4x + 9 = (2x - 3)^2$.

$4x^2 - 16x = 0$, so $x = 0$ or $x = 4$.

Check in $\sqrt{4x + 9} = 2x - 3$: $x = 4$ gives $5 \geq 0$ (valid).

$x = 0$ gives right-hand side $-3 < 0$, so it is rejected.

Answer: $x = 4$

2 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations

$\log_2 x + \log_2 y = 5$

$x - y = 31$

$\log_2(xy) = 5$, so $xy = 2^5 = 32$

$x = y + 31$: $y^2 + 31y - 32 = 0$

$(y - 1)(y + 32) = 0$; $y > 0$ so $y = 1$

Then $x = y + 31$

Answer: $x = 32$, $y = 1$

3 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|4x - 3| > -2x + 6$.

Boundary values: $4x - 3 = -2x + 6$ gives $x = \frac{3}{2}$;

$4x - 3 = -(-2x + 6)$ gives $x = -\frac{3}{2}$.

Also the corner of $|4x - 3|$ is at $x = \frac{3}{4}$.

Testing each region (check the graph of each side) gives the solution set.

Answer: $x < -\frac{3}{2}$ or $x > \frac{3}{2}$

4 Rough slope: up then back down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 15 m s^{-1} up a line of greatest slope of a rough plane inclined at α to the horizontal, where $\sin\alpha = \frac{4}{5}$ and $\mu = 0.2$.

Find (a) the distance it travels up the plane, to 3 s.f., (b) its speed on returning to its starting point. Take $g = 9.8$ m s^{-2}.

$\cos\alpha = \frac{3}{5}$. Up the plane: $R = mg\cos\alpha$, so $a_{up} = g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{4}{5} + 0.2 \times \frac{3}{5}) = 9.016$

$0 = 15^2 - 2 \times 9.016 \times d$, so $d = \frac{225}{18.032} = 12.48$

Down the plane: $a_{down} = g(\sin\alpha - \mu\cos\alpha) = 9.8(\frac{4}{5} - 0.2 \times \frac{3}{5}) = 6.664$

$v^2 = 2 \times 6.664 \times 12.478 = 166.3$, so $v = 12.9$

Answer: (a) 12.5 m (b) 12.9 m s^{-1}

5 Without replacement: find the number of counters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 5 red counters and some grey counters, n in total.

Two are taken without replacement. P(both red) = $\frac{5}{33}$. Find n.

P(both red) = $\frac{5}{n} \times \frac{4}{(n-1)} = \frac{20}{n(n-1)}$

So $n(n - 1) = 20 \div \frac{5}{33} = 132$

$n^2 - n - 132 = 0$, so $(n - 12)(n + 11) = 0$

$n > 0$, so $n = 12$

Answer: $n = 12$

Day 161 **A Level Maths: S Tier 10-a-day: Model Solutions**

6 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = t^2 - 2t$, $y = 3t^3 + t$.
Find the equation of the tangent at the point where $t = 2$,
in the form $y = mx + c$.

$dx/dt = 2t - 2$, $dy/dt = 9t^2 + 1$

At $t = 2$: $dy/dx = \frac{37}{2} = \frac{37}{2}$; point $(0, 26)$

$y - 26 = \frac{37}{2}(x - 0)$

Answer: $\mathbf{y = \frac{37}{2}x + 26}$

7 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of a parcel is $N(115, 10^2)$ kg. A parcel is 'heavy' if it exceeds 131.
(a) Find the probability that a parcel is heavy.
(b) 11 parcels are chosen at random. Find the probability that at least 4 are heavy.

(a) $P(X > 131) = P(Z > 1.600) = 0.05480$
(b) Let Y = number heavy, $Y \sim B(11, 0.05480)$
$P(Y \geq 4) = 1 - P(Y \leq 3) = 0.00218$
Answer: (a) 0.0548 (b) 0.00218

8 Periodic recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_1 = -1$ and

$u_{n+1} = \frac{2u_n + 4}{2u_n - 2}$.

Show that the sequence is periodic and find

$\sum_{r=1}^{60} u_r$.

$u_1 = -1$, $u_2 = -\frac{1}{2}$, $u_3 = -1$.

$u_3 = u_1$, so the sequence repeats with period 2.

One period sums to $-\frac{3}{2}$.

$60 = 2 \times 30 + 0$: 30 full periods plus the first 0 terms.
Answer: $\mathbf{-45}$

9 Trigonometric integrals with identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use a suitable trigonometric identity to find the exact value of

$\int_0^{\pi/2} \sin 3x \cos x \, dx$

Use $2 \sin A \cos B \equiv \sin(A + B) + \sin(A - B)$
Integrate term by term
Substitute the limits

Answer: $\mathbf{\frac{1}{2}}$

10 Block on a plank: limiting slip (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A plank of mass 2 kg lies on a smooth horizontal table, with a block of mass 4 kg on it ($\mu = 0.5$ between them). A horizontal force P N acts on the plank.

(a) Find the greatest P for which the block does not slip. (b) Find the accelerations of block and plank when P = 44.1. Take $g = 9.8$ m s^{-2}.

No slipping: common acceleration $a = \frac{P}{6}$. The block is accelerated only by friction: $F = 4a \leq \mu mg$

So $a \leq \mu g = 4.9$, and $P_{max} = 6 \times 4.9 = 29.4$

$P = 44.1 > 29.4$, so the block slips. Block: $a = \mu g = 4.9$

Plank: $44.1 - 19.6 = 2a$, so $a = 12.25$

Answer: (a) 29.4 N (b) block 4.9 m s^{-2}, plank 12.25 m s^{-2}