Day 16 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Stone thrown from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 22 m s^{-1} from the top of a cliff. The top of the cliff is 44.4 m above the sea.
Find (a) the time taken for the stone to reach the sea, (b) the total distance travelled by the stone, to 3 s.f. Take g = 9.8 m s^{-2}.

Take upwards as positive: $-44.4 = 22t - 4.9t^2$, so $4.9t^2 - 22t - 44.4 = 0$

$t = \frac{22 + \sqrt{(22^2 + 19.6 \times 44.4)}}{9.8} = \frac{22 + 36.8}{9.8} = 6$

Height risen above the cliff top = $\frac{22^2}{(2 \times 9.8)} = 24.7$ m

Total distance = $2 \times 24.7 + 44.4$

Answer: (a) 6 s (b) 93.8 m

2 Quadratic sequence from recurrence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = u_n + 2n$, $u_1 = 0$.
(a) Show that $u_n = an^2 + bn + c$ and find a, b, c.
(b) Find the least n for which $u_n > 1260$.

$u_n = u_1 + \sum_{k=1}^{n-1}(2k) = 0 + 2 \times \frac{(n-1)n}{2} + 0(n-1).$

$= n^2 - n.$

So a = 1, b = −1, c = 0.

$u_{36} = 1260 \leq 1260$ and $u_{37} = 1332 > 1260$.

Answer: (a) a = 1, b = −1, c = 0 (b) n = 37

3 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $6\text{cosec}^2\theta - 5\cot\theta - 12 = 0$
for $0 \leq \theta < 360°$, giving answers to 1 d.p.

Use $\text{cosec}^2\theta \equiv 1 + \cot^2\theta$: $6\cot^2\theta - 5\cot\theta - 6 = 0$

$(2\cot\theta - 3)(3\cot\theta + 2) = 0$

$\cot\theta = \frac{3}{2}$ or $\cot\theta = -\frac{2}{3}$

$\tan\theta = \frac{2}{3}$ or $-\frac{3}{2}$, so $\theta = 33.7°$ and $123.7°$ (+180°)

Answer: θ = 33.7°, 123.7°, 213.7°, 303.7°

4 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows y = f(x), made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of
$y = 3f(-2x - 4) - 4$.

Inside f: replace x by $-2x - 4$, so x-coordinates become $(x + 4) \div (-2)$.
Outside: y-coordinates become $3 \times y - 4$.
A: (1, −10) B: (0, −7)
C: (−2, −1) D: (−5, 8)
Answer: A′(1, −10), B′(0, −7), C′(−2, −1), D′(−5, 8)

5 One-tailed binomial test with data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A seed supplier says that 40% of its seeds germinate. 20 seeds are planted, and 4 germinate.
Test, at the 1% significance level, the claim that the proportion is lower.
Give your conclusion in context.

H_0: p = 0.4, H_1: p < 0.4. Let X ~ B(20, 0.4)

$P(X \leq 4) = 0.0510$

0.0510 is greater than 1%, so do not reject H_0

There is insufficient evidence that the proportion is lower than the claimed 0.4

Answer: Do not reject H_0: p-value 0.0510 > 1%

6 Non-uniform rod: centre of mass and tilting (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB of length 6 m and mass 5 kg rests on supports at C and D, where AC = 2 m and AD = 4 m, with $R_C = \frac{2}{3} R_D$.

Find (a) the distance of the centre of mass from A. A particle of mass m kg at B puts the rod on the point of tilting about D. (b) Find m.

$R_C = \frac{2}{3} R_D$. Moments about A: $R_C \times 2 + R_D \times 4 = 5\,g \times AG$

$R_D(\frac{2}{3} \times 2 + 4) = 5\,g \times AG$ with $R_D(1 + \frac{2}{3}) = 5\,g$, so $AG = \frac{\frac{2}{3} \times 2 + 4}{\frac{5}{3}} = 3.2$

Tilting about D: $R_C = 0$. Moments about D: $m\,g\,(6 - 4) = 5\,g\,(4 - 3.2)$

$m = \frac{5 \times 0.8}{2} = 2$

Answer: (a) 3.2 m (b) 2

7 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = \ln y + 3y^2$.

Find dy/dx and d^2y/dx^2 when $y = 1$.

$dx/dy = 6y + \frac{1}{y} = 7$ when $y = 1$, so $dy/dx = \frac{1}{dx/dy} = \frac{1}{7}$

$d^2x/dy^2 = 5$ when $y = 1$

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{5}{7^3}$

Answer: $dy/dx = \frac{1}{7}$, $d^2y/dx^2 = -\frac{5}{343}$

8 Conditional probability with a normal variable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A battery's life, in hours, is $N(74, 6^2)$.

Given that it exceeds 80, find the probability that it exceeds 86.

Required $= P(X > 86 \mid X > 80) = \frac{P(X > 86)}{P(X > 80)}$

$P(X > 86) = P(Z > 2.000) = 0.02275$

$P(X > 80) = P(Z > 1.000) = 0.1587$

$= \frac{0.02275}{0.1587} = 0.143$

Answer: 0.143

9 Remainder theorem (quadratic divisor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when x^7 is divided by $(x - 1)(x - 3)$.

$x^7 = (x - 1)(x - 3)Q(x) + px + q$.

$x = 1$: $1 = p + q$.

$x = 3$: $2187 = 3p + q$.

Subtract: $-2p = -2186$, so $p = 1093$ and $q = -1092$.

Answer: $1093x - 1092$

10 Area under a parametric curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2$, $y = 2t(6 - t)$,
for $0 \leq t \leq 6$. The curve meets the x-axis at O and at A.
Find the area enclosed between the curve and the x-axis.

dx/dt = 6t; x from 0 to 108 as t goes from 0 to 6

Area = $\int_0^6 y \frac{dx}{dt} dt = \int_0^6 12t^2(6 - t)\, dt$

$= 12[\frac{6t^3}{3} - \frac{t^4}{4}]$

Answer: 1296