Day 155 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Velocity-time graph with a return (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the velocity-time graph of a particle P moving on a straight line (velocity in m s^{-1}, time in s). At time 9 s, P is back at its starting point.

Find (a) the value of W, (b) the greatest distance of P from its starting point.

Net area = 0 (back at start): ½ × 2 × 6 + ½(6 − W)(4) − W(3) = 0

6 + 12 − 2W − 3W = 0, so 5W = 18

W = 3.6

v = 0 where the line from (2, 6) to (6, −3.6) crosses the axis: $t = 2 + 4 \times \frac{6}{9.6} = 4.5$

Greatest distance = area of triangle = ½ × 4.5 × 6

Answer: (a) W = 3.6 (b) 13.5 m

2 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 34 and upper quartile 51. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.

(a) Find the range of values that are not outliers.

(b) 15 observations have mean 46. The largest is 87, an outlier, and it is removed. Find the new mean.

IQR = 51 − 34 = 17, so 1.5 × IQR = 25.5

Lower limit = 34 − 25.5 = 8.5, upper limit = 51 + 25.5 = 76.5

Σx = 15 × 46 = 690, so without 87: Σx = 603

New mean = $\frac{603}{14}$

Answer: (a) 8.5 to 76.5 (b) 43.07

3 Exponential growth and decay models (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population of bacteria is modelled by $P = P_0e^{kt}$, t in hours.

Initially P = 200 and after 5 hours P = 800.

Find k, and the time for the population to reach 2400, to 3 s.f.

$P_0 = 200$. $200\,e^{5k} = 800$ so $e^{5k} = 4$

$k = \frac{1}{5} \ln 4 = 0.277$

$200\,e^{kt} = 2400$ so $e^{kt} = 12$, $t = \frac{1}{k} \ln 12$

t = 8.96 hours

Answer: k = 0.277, t = 8.96 hours

4 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{x + 8} = x + 6$.

Square both sides: $x + 8 = (x + 6)^2$.

$x^2 + 11x + 28 = 0$, so x = −7 or x = −4.

Check in $\sqrt{x + 8} = x + 6$: x = −4 gives 2 ≥ 0 (valid).

x = −7 gives right-hand side −1 < 0, so it is rejected.

Answer: x = −4

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5 Projectile from a cliff (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the top of a vertical cliff, 58.8 m high, at 24.5 m s^{-1} at angle α below the horizontal ($\tan\alpha = \frac{4}{3}$), and lands on horizontal ground.
Find (a) the time of flight and the distance from the cliff, (b) the speed and direction on landing. Take g = 9.8 m s^{-2}.

Vertically (up positive): $-58.8 = -19.6\,t - 4.9t^2$, so $4.9t^2 + 19.6t - 58.8 = 0$
t = 2 s (the positive root)
Horizontal distance = 14.7 × 2 = 29.4 m
On landing: $v_x = 14.7$, $v_y = -19.6 - 9.8 \times 2 = -39.2$
speed = $\sqrt{(14.7^2 + 39.2^2)} = 41.9$, tan angle = $\frac{39.2}{14.7}$, angle = 69.4° below the horizontal

Answer: (a) 2 s, 29.4 m (b) 41.9 m s^{-1} at 69.4° below the horizontal

6 Independence from a Venn diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities; A and B are independent.
Find x and y.

P(A) = 1x + x = 2x, P(B) = x + y, $P(A \cap B) = x$

Independent: $x = 2x(x + y)$, and $x \neq 0$, so $x + y = \frac{1}{2}$

Total probability: $x + x + y + 0.4 = 1$, so $x + (x + y) = 0.6$

$x = 0.6 - \frac{1}{2} = 0.1$

$x = \frac{1}{10}$, $y = \frac{1}{2} - x = \frac{2}{5}$

Answer: $x = \frac{1}{10}$, $y = \frac{2}{5}$

7 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 596
inclusive that are divisible by 3 or by 4, but not by both.

Multiples of 3: 198 terms (3 to 594): sum = 59103.
Multiples of 4: 149 terms: sum = 44700.
Multiples of both, i.e. of 12: 49 terms: sum = 14700.
Required sum = 59103 + 44700 − 2 × 14700.
Answer: 74403

8 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of
$\sin(\sin^{-1}(\frac{7}{25}) + \cos^{-1}(\frac{4}{5}))$.

Let $A = \sin^{-1}(\frac{7}{25})$ and $B = \cos^{-1}(\frac{4}{5})$, both acute

$\sin A = \frac{7}{25}$, $\cos A = \frac{24}{25}$, $\cos B = \frac{4}{5}$, $\sin B = \frac{3}{5}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{7}{25} \times \frac{4}{5} + \frac{24}{25} \times \frac{3}{5}$

Answer: $\frac{4}{5}$

9 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(−6, −5) and B(1, 12). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c - (-6))^2 + 25 = (c - 1)^2 + 144$.
Expand and solve: 14c = 84, so c = 6.
Area = $\frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$
$= \frac{1}{2}|(-12) \times 12 - (-5) \times (-5)| = \frac{169}{2}$.

Answer: C(6, 0); area $\frac{169}{2}$

10 Lines in 3D: intersection and angle (CIE (9709); Further Maths on other boards)

Line l_1: $\mathbf{r} = \begin{pmatrix} -8 \\ -6 \\ -5 \end{pmatrix} + s\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$,

line l_2: $\mathbf{r} = \begin{pmatrix} -1 \\ -5 \\ -1 \end{pmatrix} + t\begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$.

Show whether the lines meet, giving the point of intersection if they do, and find the acute angle between them to 1 d.p.

Equate components: −8 + 2s = −1 + 3t; −6 + 2s = −5 − 3t; −5 + s = −1 + 2t
Solving the first two: s = 2, t = −1; the third equation is satisfied
Intersection: (−4, −2, −3)
$\cos\varphi = \frac{|2|}{\sqrt{9}\sqrt{22}} = 0.1421$, φ = 81.8°

Answer: Lines meet at (−4, −2, −3); angle 81.8°