Day 153 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-2\mathbf{i} + 8\mathbf{j})$ N, $(4\mathbf{i} + k\mathbf{j})$ N and $(2\mathbf{i} - 3\mathbf{j})$ N act on a particle of mass 3 kg. The resultant is parallel to $(4\mathbf{i} - 3\mathbf{j})$. Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant = $(4)\mathbf{i} + (5 + k)\mathbf{j}$

Parallel to $(4\mathbf{i} - 3\mathbf{j})$: $\frac{5+k}{4} = \frac{-3}{4}$, so $k = -8$

R = $(4\mathbf{i} - 3\mathbf{j})$ N, so $|R| = \sqrt{(4^2 + -3^2)} = 5.000$

a = |R| ÷ 3

Answer: (a) $k = -8$ (b) 1.67 m s^{-2}

2 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cot(2\theta - 20°) = 1$
for $0° \leq \theta \leq 360°$.

Let $u = 2\theta - 20°$; then $\cot u = 1$

Base solutions: $u = 45°$ (+ multiples of 180°)

$\theta = (u + 20°) \div 2$, keeping those in range

Answer: θ = 32.5°, 122.5°, 212.5°, 302.5°

3 Binomial approximation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(16 - x)^{-\frac{1}{2}}$ up to and including the term in x^2.

(b) Use x = 0.01 to estimate $\frac{1}{\sqrt{15.99}}$ to 5 decimal places.

(a) $(16 - x)^{-\frac{1}{2}} = \frac{1}{4}(1 - \frac{1}{16}x)^{-\frac{1}{2}}$.

$= \frac{1}{4} + \frac{1}{128}x + \frac{3}{8192}x^2 + ...$

(b) $16 - x$ with x = 0.01 gives 15.99.

Substitute: $\frac{1}{4} + \frac{1}{128}(0.01) + \frac{3}{8192}(0.01)^2$.

= 0.25008 (5 d.p.).

Answer: (a) $\frac{1}{4} + \frac{1}{128}x + \frac{3}{8192}x^2$ (b) 0.25008

4 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of

$\sin(\sin^{-1}(\frac{8}{17}) + \cos^{-1}(\frac{3}{5}))$.

Let $A = \sin^{-1}(\frac{8}{17})$ and $B = \cos^{-1}(\frac{3}{5})$, both acute

$\sin A = \frac{8}{17}$, $\cos A = \frac{15}{17}$, $\cos B = \frac{3}{5}$, $\sin B = \frac{4}{5}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{8}{17} \times \frac{3}{5} + \frac{15}{17} \times \frac{4}{5}$

Answer: $\frac{84}{85}$

5 Combined mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Class R has 24 students with mean mark 31 and standard deviation 7. Class S has 30 students with mean mark 30 and standard deviation 9.
(a) Find the mean mark of all 54 students.
(b) Find the standard deviation of all 54 students.

Σx for R = 24 × 31 = 744, Σx for S = 30 × 30 = 900

Combined mean = $\frac{1644}{54}$ = 30.4

$\Sigma x^2 = n(\sigma^2 + \text{mean}^2)$: 24(49 + 961) + 30(81 + 900) = 53670

Variance = $\frac{53670}{54} - 30.444^2$ = 67.025, so σ = 8.187

Answer: (a) 30.4 (b) 8.19

6 Rough slope: up then back down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 18 m s^{-1} up a line of greatest slope of a rough plane inclined at α to the horizontal, where $\sin\alpha = \frac{3}{5}$ and $\mu = 0.4$.

Find (a) the distance it travels up the plane, to 3 s.f., (b) its speed on returning to its starting point. Take g = 9.8 m s^{-2}.

$\cos\alpha = \frac{4}{5}$. Up the plane: R = mg cos α, so $a_{up} = g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{3}{5} + 0.4 \times \frac{4}{5}) = 9.016$

$0 = 18^2 - 2 \times 9.016 \times d$, so $d = \frac{324}{18.032} = 17.97$

Down the plane: $a_{down} = g(\sin\alpha - \mu\cos\alpha) = 9.8(\frac{3}{5} - 0.4 \times \frac{4}{5}) = 2.744$

$v^2 = 2 \times 2.744 \times 17.968 = 98.61$, so v = 9.93

Answer: (a) 18.0 m (b) 9.93 m s^{-1}

7 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x - 1| \geq |-4x + 5|$.

Square both sides: $(2x - 1)^2 \geq (-4x + 5)^2$.
$(6x - 6)(-2x + 4) \geq 0$.
Critical values x = 1 and x = 2.
The quadratic has negative coefficient of x^2, so choose the inner region.

Answer: $1 \leq x \leq 2$

8 Circle through three points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through P(3, −2), Q(−1, 6)
and R(0, 7). Find its equation.

Centre is on the perpendicular bisector of each chord.
Bisector of PQ: $-4x + 8y = 12$.
Bisector of QR: $x + y = 6$.
Solving: centre (3, 3).
$r^2 = (3 - 3)^2 + (-2 - 3)^2 = 25$.

Answer: $(x - 3)^2 + (y - 3)^2 = 25$

9 Poisson hypothesis test (CIE)

Faults in a cable occur at random at a mean rate of 2 per week. After a modification, 8 faults are found in 1 week.
Test at the 10% level whether the rate of faults has increased.

H_0: $\lambda = 2$ per week, H_1: $\lambda > 2$ per week
Over 1 week: X ~ Po(2)
$P(X \geq 8) = 1 - P(X \leq 7) = 0.00110$
0.00110 < 10%, so reject H_0
There is sufficient evidence that the rate has increased

Answer: Reject H_0 (p-value 0.00110)

Day 153 **A Level Maths: S Tier 10-a-day: Model Solutions**

10 Area under a parametric curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2\sin t$, $y = 6\cos t$, for $0 \le t \le \frac{\pi}{2}$. Find the exact area bounded by the curve and the coordinate axes.

$dx/dt = 2\cos t$

Area $= \int_0^{\pi/2} 6\cos t \times 2\cos t \, dt = 12\int_0^{\pi/2} \cos^2 t \, dt$

$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$: integral $= 12 \times \frac{\pi}{4}$

Answer: 3π