Day 152 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas, CIE (small angle))

θ is small and measured in radians. Use small angle approximations to find the value that $\frac{\theta \sin 3\theta}{1 - \cos 5\theta}$ approaches as $\theta \to 0$.

$\sin 3\theta \approx 3\theta$ and $1 - \cos 5\theta \approx \frac{25}{2}\theta^2$

Expression $\approx \frac{3\theta^2}{\frac{25}{2}\theta^2} = \frac{6}{25}$

Answer: $\frac{6}{25}$

2 Particle in equilibrium on two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P of weight 63 N hangs in equilibrium from two light strings AP and BP attached to a horizontal ceiling at A and B (see diagram), where $\tan\alpha = \frac{4}{3}$ and $\tan\beta = \frac{3}{4}$.

Find (a) the tension in each string, (b) the greatest weight P could have if neither tension may exceed 86.4 N.

Horizontally: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_1 \times \frac{3}{5} = T_2 \times \frac{4}{5}$

Vertically: $T_1 \sin\alpha + T_2 \sin\beta = 63$, and $T_2 = \frac{3}{4} T_1$

$T_1(\frac{4}{5} + \frac{3}{4} \times \frac{3}{5}) = 63$, so $T_1 = 50.4$ and $T_2 = 37.8$

The tension in AP is the larger: it is 50.4 when W = 63

Tensions are proportional to W, so $W_{max} = 63 \times \frac{86.4}{50.4} = 108$

Answer: (a) AP: 50.4 N, BP: 37.8 N (b) 108 N

3 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations

$\lg x + \lg y = 1$

$x - y = 9$

$\lg(xy) = 1$, so $xy = 10^1 = 10$

$x = y + 9$: $y^2 + 9y - 10 = 0$

$(y - 1)(y + 10) = 0$; $y > 0$ so $y = 1$

Then $x = y + 9$

Answer: x = 10, y = 1

4 Cumulative distribution function (CIE)

The cdf of X is $F(x) = 0$ for $x < 0$, $F(x) = \frac{x^2}{25}$ for $0 \le x \le 5$, and $F(x) = 1$ for $x > 5$.

(a) Find the pdf of X.

(b) Find E(X).

(c) Find P(1.25 < X < 2.5) and the median, to 3 significant figures.

(a) $f(x) = F'(x) = \frac{2}{25} x$ for $0 \le x \le 5$

(b) $E(X) = \int x f(x)\, dx$ from 0 to 5 $= \frac{10}{3}$

(c) $P = F(2.5) - F(1.25) = 0.1875$

Median m: $F(m) = 1/2$, so $m = 5 \times (1/2)^{1/2}$

$m = 3.536$

Answer: (a) f(x) = $\frac{2}{25} x$ (b) $\frac{10}{3}$ (c) 0.188, median 3.54

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5 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{x + 12} = x + 6$.

Square both sides: $x + 12 = (x + 6)^2$.

$x^2 + 11x + 24 = 0$, so $x = -8$ or $x = -3$.

Check in $\sqrt{x + 12} = x + 6$: $x = -3$ gives $3 \geq 0$ (valid).

$x = -8$ gives right-hand side $-2 < 0$, so it is rejected.

Answer: $x = -3$

6 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2 - t$, $y = t^3 - 4t$.
Find the equation of the tangent at the point where $t = 1$,
in the form $y = mx + c$.

$dx/dt = 4t - 1$, $dy/dt = 3t^2 - 4$

At $t = 1$: $dy/dx = \frac{-1}{3} = -\frac{1}{3}$; point $(1, -3)$

$y - -3 = (-\frac{1}{3})(x - 1)$

Answer: $y = -\frac{1}{3}x - \frac{8}{3}$

7 Self-inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{kx + 9}{x - 3}$ for $x \neq 3$. Given that f is self-inverse, find k, and hence solve $f(x) = x$.

Let $y = f(x)$: $y(x - 3) = kx + 9$, so $x(y - k) = 3y + 9$.

$f^{-1}(x) = \frac{3x + 9}{x - k}$.

$f = f^{-1}$ requires $k = 3$.

$f(x) = x$: $3x + 9 = x(x - 3)$, so $x^2 - 6x - 9 = 0$.

$x = 3 \pm \sqrt{18} = 3 \pm 3\sqrt{2}$.

Answer: $k = 3$; $x = 3 \pm 3\sqrt{2}$

8 Projectile through a given point: two angles (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 35 m s^{-1}, at angle θ above the horizontal. It passes through the point P, 70 m horizontally and 38 m vertically from O.

Find the two possible values of $\tan\theta$, and the two possible values of θ to 1 d.p. Take $g = 9.8$ m s^{-2}.

Horizontal: $x = 35\cos\theta \times t$. Vertical: $y = 35\sin\theta \times t - 4.9t^2$

Eliminating t: $y = x\tan\theta - \frac{9.8x^2}{2u^2}(1 + \tan^2\theta)$

$\frac{9.8 \times 70^2}{2 \times 35^2} = 19.6$, so $38 = 70\tan\theta - 19.6(1 + \tan^2\theta)$

$19.6\tan^2\theta - 70\tan\theta + 57.6 = 0$

$\tan\theta = \frac{9}{7}$ or $\frac{16}{7}$, so $\theta = 52.1°$ or $66.4°$

Answer: $\tan\theta = \frac{9}{7}$ or $\frac{16}{7}$; $\theta = 52.1°$ or $66.4°$

9 Perpendicular lines and area (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the diagram, line l_1 passes through A and B.
Line l_2 is perpendicular to l_1 at B and meets the
x-axis at C. Find the coordinates of C and the area of triangle ABC.

Gradient of $l_1 = \frac{6}{4-13} = -\frac{2}{3}$.

Gradient of $l_2 = \frac{3}{2}$. Equation: $y - 6 = \frac{3}{2}(x - 4)$.

y = 0: x = 0, so C(0, 0).

AB = $3\sqrt{13}$, BC = $2\sqrt{13}$: area = ½ × AB × BC.

Answer: C(0, 0); area 39

10 Symmetric limits of a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(98, 144^2)$. Find the value k such that $P(98 - k < X < 98 + k) = 0.70$.
Give k to 3 significant figures.

The interval is symmetrical about the mean, so each tail has probability 0.150

$P(X < 98 - k) = 0.150$, so $\frac{-k}{12} = -1.0364$

k = 12 × 1.0364 = 12.44

Answer: k = 12.4