**Day 150** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 1 PMCC from summary data and a hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

For 20 pairs of values, $S_{xx} = 144$, $S_{yy} = 256$ and $S_{xy} = 131$.

(a) Calculate the product moment correlation coefficient r.

(b) The critical value for a one-tailed test at the 5% level with n = 20 is 0.3783. Carry out the test of $H_0$: $\rho = 0$ against $H_1$: $\rho > 0$.

$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}} = \frac{131}{\sqrt{(144 \times 256)}} = \frac{131}{192} = 0.6823$

$H_0$: $\rho = 0$, $H_1$: $\rho > 0$

$|r| = 0.6823 >$ the critical value 0.3783

Reject $H_0$: there is sufficient evidence of positive correlation between the variables

**Answer: (a) r = 0.682 (b) reject $H_0$**

## 2 Surds: rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{2 - 3\sqrt{7}}{6 - 2\sqrt{7}}$ in the form $p + q\sqrt{7}$, where p and q are rational numbers.

Multiply top and bottom by the conjugate $(6 + 2\sqrt{7})$.

Numerator: $(2 - 3\sqrt{7})(6 + 2\sqrt{7}) = -30 - 14\sqrt{7}$.

Denominator: $6^2 - 2^2(7) = 8$.

So the fraction = $\frac{-30 - 14\sqrt{7}}{8}$.

**Answer: $-\frac{15}{4} - \frac{7}{4}\sqrt{7}$**

## 3 Overtaking with a delayed start (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O travelling at a constant speed of 12.8 m $s^{-1}$. 9 s later, car B leaves O from rest with constant acceleration 2.5 m $s^{-2}$, travelling in the same direction.

B overtakes A at the point X. Find (a) the time B takes to reach X, (b) the distance OX, (c) the speed of B at X.

Let B travel for t s. A: distance = 12.8(t + 9); B: distance = ½ × 2.5 × $t^2$

Equate: $2.5t^2 - 25.6t - 230.4 = 0$, so $(t - 16)(2.5t + 14.4) = 0$

t = 16 (the other root is negative)

OX = 2.5 × $16^2$ / 2 = 320 m, and speed = 2.5 × 16 = 40

**Answer: (a) 16 s (b) 320 m (c) 40 m $s^{-1}$**

## 4 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has its centre in the first quadrant and touches both the x-axis and the y-axis. It passes through the point (8, 9). Find the two possible equations of the circle.

Centre (r, r), radius r: $(x - r)^2 + (y - r)^2 = r^2$.

Substitute (8, 9): $(8 - r)^2 + (9 - r)^2 = r^2$.

$r^2 - 34r + 145 = 0$.

$(r - 5)(r - 29) = 0$, so r = 5 or r = 29.

**Answer: $(x - 5)^2 + (y - 5)^2 = 25$ or $(x - 29)^2 + (y - 29)^2 = 841$**

## 5 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = \ln y + 4y^2$.

Find dy/dx and $d^2y/dx^2$ when y = 1.

$dx/dy = 8y + \frac{1}{y} = 9$ when y = 1, so $dy/dx = \frac{1}{dx/dy} = \frac{1}{9}$

$d^2x/dy^2 = 7$ when y = 1

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{7}{9^3}$

**Answer: $dy/dx = \frac{1}{9}$, $d^2y/dx^2 = -\frac{7}{729}$**

## 6 Without replacement: find the number of counters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 2 blue counters and some grey counters, n in total.

Two are taken without replacement. P(at least one blue) = $\frac{3}{5}$. Find n.

P(neither blue) = $\frac{(n-2)}{n} \times \frac{(n-3)}{(n-1)} = \frac{2}{5}$

$\frac{2}{5}\,n(n-1) = (n-2)(n-3)$

Rearranging gives a quadratic in n whose roots are n = 6 and one other value

The other root is $\frac{5}{3}$ so is rejected (not a valid number of counters)

n = 6

**Answer: n = 6**

## 7 Volumes of revolution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region bounded by y = sin x + 3cos x, the x-axis and the lines x = 0 and x = π/2 is rotated through 2π about the x-axis. Find the exact volume of the solid formed.

$V = \pi\int_0^{\pi/2} y^2\,dx = \pi\int_0^{\pi/2} (\sin x + 3\cos x)^2\,dx$

Simplify the integrand, then integrate

$V = \pi \times (3 + \frac{5}{2}\pi)$

**Answer: $\frac{1}{2}\pi(6 + 5\pi)$**

## 8 Least force to drag a box (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of weight 143 N is pulled at constant speed along rough horizontal ground ($\mu = \frac{5}{12}$) by a force P N at angle θ above the horizontal.

(a) Find P when $\tan\theta = \frac{8}{15}$. (b) Find the least possible value of P as θ varies, and the corresponding θ to 1 d.p.

Vertical: R + P sin θ = 143. Horizontal: P cos θ = μR = $\frac{5}{12}(143 - P\sin\theta)$

$P(\cos\theta + \frac{5}{12}\sin\theta) = \frac{5}{12} \times 143$, so $P = \frac{715}{12} \div (\cos\theta + \frac{5}{12}\sin\theta)$

(a) $\cos\theta = \frac{15}{17}$, $\sin\theta = \frac{8}{17}$: $P = \frac{715}{12} \div \frac{55}{51} = 55.25$

(b) P is least when $\cos\theta + \frac{5}{12}\sin\theta$ is greatest. Its greatest value is $\sqrt{(1 + (\frac{5}{12})^2)} = \frac{13}{12}$,

which occurs when $\tan\theta = \frac{5}{12}$. Then $P = \frac{715}{12} \div \frac{13}{12} = 55$ and θ = 22.6°

**Answer: (a) 55.25 N (b) 55 N at θ = 22.6°**

## 9 Remainder theorem (quadratic divisor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

When a polynomial f(x) is divided by (x + 2) the remainder is −17; when divided by (x − 2) the remainder is 3. Find the remainder when f(x) is divided by (x + 2)(x − 2).

f(x) = (x + 2)(x − 2)Q(x) + px + q.
f(−2) = −17: −2p + q = −17.
f(2) = 3: 2p + q = 3.
Subtracting: −4p = −20, so p = 5, q = −7.
**Answer: 5x − 7**

## 10 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 + 2xy + 5y^2 = 144$.
Find the coordinates of the points on C where the tangent is parallel to the x-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + 2y}{2x + 10y}$

Horizontal tangent when $2x + 2y = 0$, so $y = -\frac{2x}{2}$

Substitute into the equation of C: $x^2 + (2xy) + (5y^2) = 144$
Solve for the remaining coordinate
**Answer: $(6, -6)$ and $(-6, 6)$**