Day 15 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 600 kg, carrying a passenger of mass 90 kg, is supported by a vertical cable. The lift is moving upwards and decelerating at 0.4 m s^{-2}.

Find (a) the force the floor exerts on the passenger, (b) the tension in the cable. Take g = 9.8 m s^{-2}.

Take the direction of the acceleration as positive: it is downwards

Passenger: $90 \times 9.8 - R = 90 \times 0.4$

$R = 90 \times (9.8 - 0.4) = 846$

Lift and passenger: $690 \times 9.8 - T = (600 + 90) \times 0.4$

$T = 690 \times 9.4 = 6486$

Answer: (a) 846 N (b) 6486 N

2 Differentiating a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 5^x - x$ has one stationary point.

Find its coordinates to 3 s.f. and show that it is a minimum.

$dy/dx = 5^x \ln 5 - 1 = 0$

$5^x = \frac{1}{\ln 5}$, $x = \frac{1}{\ln 5} \ln(\frac{1}{\ln 5}) = -0.296$

$d^2y/dx^2 = 5^x(\ln 5)^2 > 0$ for all x: minimum

$y = \frac{1}{\ln 5} - 1 \times x = 0.917$

Answer: (−0.296, 0.917)

3 Tangent to a circle and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $x^2 + y^2 - 6x - 2y - 90 = 0$

passes through P(9, −7). The tangent at P meets the axes at A and B. Find the area of triangle OAB.

Centre (3, 1). Gradient of radius CP = $-\frac{4}{3}$.

Tangent gradient = $\frac{3}{4}$: $6x - 8y = 110$.

x = 0: $y = -\frac{55}{4}$. y = 0: $x = \frac{55}{3}$.

Area = ½ × $\frac{55}{3}$ × $\frac{55}{4}$.

Answer: $\frac{3025}{24}$

4 Equations quadratic in an exponential (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2^{2x} - 14 \times 2^x + 40 = 0$,

giving each answer exactly and, where not an integer, to 3 s.f.

Let $u = 2^x$: $u^2 - 14u + 40 = 0$

$(u - 4)(u - 10) = 0$, so $2^x = 4$ or $2^x = 10$

Take logarithms: $x = \log_2 4 = 2$; $x = \log_2 10 = 3.32$

Answer: $x = \log_2 4 = 2$; $x = \log_2 10 = 3.32$

5 Self-inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{kx + 4}{x - 2}$ for $x \neq 2$. Given that f is self-inverse, find k, and hence solve f(x) = x.

Let y = f(x): $y(x - 2) = kx + 4$, so $x(y - k) = 2y + 4$.

$f^{-1}(x) = \frac{2x + 4}{x - k}$.

$f = f^{-1}$ requires k = 2.

f(x) = x: $2x + 4 = x(x - 2)$, so $x^2 - 4x - 4 = 0$.

$x = 2 \pm \sqrt{8} = 2 \pm 2\sqrt{2}$.

Answer: $k = 2$; $x = 2 \pm 2\sqrt{2}$

6 Speed decaying exponentially (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. The velocity of P is v m s^{-1} and its acceleration is $-0.1v^2$ m s^{-2}. At t = 0, v = 20. Find (a) the time taken for the speed of P to halve, (b) the distance P travels in this time, to 3 s.f.

$\frac{dv}{dt} = -0.1v^2$: $\int \frac{dv}{v^2} = -0.1\int dt$, so $\frac{1}{v} = 0.1t + c$, and $\frac{1}{v} = 0.1t + \frac{1}{20}$

v = 10: $\frac{1}{v} = 0.1$, so $0.1t = 0.1 - 0.05$, giving t = 0.5

$s = \int \frac{1}{(0.1t + 1/20)}\, dt = \frac{1}{0.1} \ln(1 + 0.1 \times 20t)$

At t = 0.5: $s = \frac{1}{0.1} \ln 2$

Answer: (a) 0.5 s (b) 6.93 m

7 Triple angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE (triple angle))

Given that $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$, solve
$\cos 3\theta = 0$ for $0 < \theta < 360°$.

Substitute: $\cos\theta(4\cos^2\theta - 3 - 0) = 0$

$\cos\theta = 0$ or $\cos^2\theta = \frac{3}{4}$

$\cos\theta = 0$ gives 90°, 270°

$\cos^2\theta = \frac{3}{4}$ gives 30°, 150°, 210°, 330°

Answer: θ = 30°, 90°, 150°, 210°, 270°, 330°

8 Normal: find mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The masses of apples, in grams are normally distributed. 5% are less than 38 and 20% are greater than 69. Find the mean and standard deviation, each correct to 3 significant figures.

Let $X \sim N(\mu, \sigma^2)$. $P(X < 38) = 0.05$ gives $\frac{38 - \mu}{\sigma} = -1.6449$

$P(X > 69) = 0.20$ gives $\frac{69 - \mu}{\sigma} = 0.8416$

Subtract: $\frac{31}{\sigma} = 0.8416 - (-1.6449) = 2.4865$

$\sigma = 12.47$

$\mu = 38 - (-1.6449)(12.47) = 58.51$

Answer: μ = 58.5, σ = 12.5

9 Periodic recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_1 = 7$ and

$u_{n+1} = \frac{u_n - 2}{u_n - 1}$.

Show that the sequence is periodic and find

$\sum_{r=1}^{99} u_r$.

$u_1 = 7$, $u_2 = \frac{5}{6}$, $u_3 = 7$.

$u_3 = u_1$, so the sequence repeats with period 2.

One period sums to $\frac{47}{6}$.

99 = 2 × 49 + 1: 49 full periods plus the first 1 term.

Answer: $\frac{2345}{6}$

10 Normal approximation to a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(60, 0.25).

(a) State why a normal approximation is suitable for X.

(b) Use a normal approximation to estimate P(X ≥ 26), to 3 significant figures.

np = 15 and nq = 45, both greater than 5 (n large, p not extreme)

Y ~ N(15, 11.25) with σ = 3.354

Apply the continuity correction:

P(X ≥ 26) ≈ P(Y > 25.5) = P(Z > 3.130) = 0.000873

Answer: 0.000873