Day 149 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Double angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3\cos 2\theta - 9\sin\theta - 6 = 0$
for $0 \leq \theta < 2\pi$, giving answers in terms of π.

Use $\cos 2\theta \equiv 1 - 2\sin^2\theta$: $6\sin^2\theta + 9\sin\theta + 3 = 0$
$3(2\sin\theta + 1)(\sin\theta + 1) = 0$

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = -1$

Principal values and symmetry in $[0, 2\pi)$

Answer: $\boldsymbol{\theta = \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}}$

2 Conditional probability from given values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, $P(A) = 0.75$, $P(B) = 0.3$ and $P(A \mid B) = 0.75$.
(a) Find $P(A \cap B)$.
(b) Find $P(A \mid B')$.

$P(A \cap B) = P(A \mid B) \times P(B) = 0.75 \times 0.3 = 0.225$
$P(A \cup B) = 0.75 + 0.3 - 0.225 = 0.825$
$P(B') = 0.7$
$P(A \cap B') = P(A) - P(A \cap B) = 0.525$

$P(A \mid B') = \frac{0.525}{0.7}$

Answer: (a) 0.225 (b) $\boldsymbol{\frac{3}{4}}$

3 Beam on two supports: where can a person stand (AQA, Edexcel, OCR, MEI)

A uniform plank AB of length 8 m and mass 30 kg rests on supports at C and D, where AC = 3 m and DB = 3 m. A girl of mass 40 kg stands on it at distance x m from A.
Find the range of x for which the plank stays in equilibrium on both supports.

Equilibrium on both supports needs $R_C \geq 0$ and $R_D \geq 0$. Limiting cases: $R_C = 0$ and $R_D = 0$ in turn

Tilting about D (girl beyond D): $30(1) = 40(x - 5)$, so $x = 5 + \frac{30 \times 1}{40} = 5.75$

Tilting about C (girl before C): $30(1) = 40(3 - x)$, so $x = 3 - \frac{30 \times 1}{40} = 2.25$

Answer: $\boldsymbol{2.25 \leq x \leq 5.75}$

4 Identity then equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Show that $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$.

(b) Hence solve $\cos^4\theta - \sin^4\theta = \frac{1}{4}$ for $0 \leq \theta < 360°$ (1 d.p.).

$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = \cos 2\theta \times 1$

$\cos 2\theta = \frac{1}{4}$, so $2\theta = 75.5°, 284.5°, 435.5°, 644.5°$ $(0 \leq 2\theta < 720°)$

Halve each value

Answer: $\boldsymbol{\theta = 37.8°, 142.2°, 217.8°, 322.2°}$

5 Proof by contradiction (irrationality) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sqrt{15}$ is irrational, prove by contradiction that $\sqrt{3} + \sqrt{5}$ is irrational.

Assume $\sqrt{3} + \sqrt{5} = r$, where r is rational.
Square both sides: $3 + 5 + 2\sqrt{15} = r^2$.

So $\sqrt{15} = \frac{r^2 - 8}{2}$.

The right-hand side is rational, but $\sqrt{15}$ is irrational.
This is a contradiction, so $\sqrt{3} + \sqrt{5}$ is irrational.

Answer: Proved

6 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = x\sqrt{5x - 6}$ at the point where $x = 2$ meets the axes at A and B. Find the exact area of triangle OAB, where O is the origin.

At $x = 2$: $y = 4$. Differentiate: gradient = $\frac{9}{2}$

Tangent: $y - 4 = \frac{9}{2}(x - 2)$, so $y = \frac{9}{2}x - 5$

y-intercept -5; x-intercept $\frac{10}{9}$

Area = $\frac{1}{2} \times |-5| \times |\frac{10}{9}|$

Answer: $\frac{25}{9}$

7 Combined mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Class S has 10 students with mean mark 35 and standard deviation 4. Class Q has 15 students with mean mark 67 and standard deviation 9.

(a) Find the mean mark of all 25 students.

(b) Find the standard deviation of all 25 students.

Σx for S = $10 \times 35 = 350$, Σx for Q = $15 \times 67 = 1005$

Combined mean = $\frac{1355}{25} = 54.2$

$\Sigma x^2 = n(\sigma^2 + \text{mean}^2)$: $10(16 + 1225) + 15(81 + 4489) = 80960$

Variance = $\frac{80960}{25} - 54.200^2 = 300.76$, so $\sigma = 17.34$

Answer: (a) 54.2 (b) 17.3

8 Cubic graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the cubic $y = f(x)$, which touches the x-axis once. Find f(x), giving your answer in expanded form.

Crosses at $x = 4$ and touches at $x = -1$, so $f(x) = k(x - 4)(x + 1)^2$.

At $(0, -4)$: $-4 = k(-4)(1)^2 = -4k$, so $k = 1$.

$f(x) = (x - 4)(x + 1)^2$, then expand.

Answer: f(x) = $x^3 - 2x^2 - 7x - 4$

9 Horizontal force on a rough slope: range of values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of weight 170 N rests on a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{4}{3}$ and $\mu = 0.5$.

A horizontal force H N, in the vertical plane through a line of greatest slope, holds it in equilibrium. Find the range of possible values of H.

$\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$. Normal reaction: $R = 170\cos\alpha + H\sin\alpha$

About to slide down: $H\cos\alpha + \mu R = 170\sin\alpha$, so $H(\frac{3}{5} + 0.5 \times \frac{4}{5}) = 170(\frac{4}{5} - 0.5 \times \frac{3}{5})$

$H_{min} = 85$

About to slide up: $H\cos\alpha = 170\sin\alpha + \mu R$, so $H(\frac{3}{5} - 0.5 \times \frac{4}{5}) = 170(\frac{4}{5} + 0.5 \times \frac{3}{5})$

$H_{max} = 935$

Answer: $85 \leq H \leq 935$

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10 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{3x - 5} = x - 3$.

Square both sides: $3x - 5 = (x - 3)^2$.

$x^2 - 9x + 14 = 0$, so $x = 2$ or $x = 7$.

Check in $\sqrt{3x - 5} = x - 3$: $x = 7$ gives $4 \geq 0$ (valid).

$x = 2$ gives right-hand side $-1 < 0$, so it is rejected.

Answer: x = 7