Day 148 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Rational inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{4}{(x+1)} > \frac{3}{(x-1)}$.

Multiply by the positive quantity $(x+1)^2(x-1)^2$:
$4(x-(-1))(x-1)^2 > 3(x-1)(x-(-1))^2$.
Move to one side and factorise: $(x-(-1))(x-1)(x-7) > 0$.
Critical values: -1, 1, 7. Test the four regions.
Answer: $-1 < x < 1$ or $x > 7$

2 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A shell is projected from level ground with horizontal and vertical velocity components 14.7 and 63.7 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take g = 9.8 m s^{-2}.

At 45° to the horizontal, $|v_y| = v_x = 14.7$
Moving upwards: $63.7 - 9.8t = 14.7$, so $t = 5$
Moving downwards: $63.7 - 9.8t = -14.7$, so $t = 8$
Horizontal distance $= 14.7 \times (8 - 5)$
Answer: (a) 5 s and 8 s (b) 44.1 m

3 Coded data: mean, standard deviation, Σx^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 40 observations x, $\Sigma(x-50) = 20$ and $\Sigma(x-50)^2 = 970$.
(a) Find the mean of x.
(b) Find the standard deviation of x.
(c) Find the value of Σx^2.

Mean of $(x-50) = \frac{20}{40} = 0.5$, so mean of $x = 50.5$

Variance $= \frac{970}{40} - (0.5)^2 = 24$

$\sigma = \sqrt{24}$
$\Sigma x^2 = \Sigma(x-50)^2 + 100\,\Sigma(x-50) + 40 \times 50^2$
$= 970 + 2000 + 100000$
Answer: (a) 50.5 (b) 4.90 (c) 102970

4 Proof by contradiction (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove by contradiction that $\log_6 2$ is irrational.

Assume $\log_6 2 = \frac{m}{n}$, m, n positive integers.

Then $6^{\frac{m}{n}} = 2$, so $6^m = 2^n$.
Prime factors of 6^m and 2^n must be identical,
but 3 divides 6^m but 3 does not divide 2^n.
This is a contradiction, so $\log_6 2$ is irrational.
Answer: Proved

5 Addition formulae with exact values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is acute and B is acute. $\cos A = \frac{21}{29}$ and $\sin B = \frac{20}{29}$.
Find the exact values of $\sin(A-B)$ and $\sin(A+B)$.

$\cos A = \frac{21}{29}$, $\sin A = \frac{20}{29}$; $\sin B = \frac{20}{29}$, $\cos B = \frac{21}{29}$
$\sin(A-B) = \sin A \cos B - \cos A \sin B = 0$
$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{840}{841}$
Answer: $\sin(A-B) = 0$, $\sin(A+B) = \frac{840}{841}$

6 Pdf of the form a + bx with a given mean (CIE)

The pdf of X is f(x) = a + bx for $0 \leq x \leq 2$, and 0 otherwise, where a and b are constants. E(X) = $\frac{17}{15}$.

(a) Find a and b.

(b) Find P(X < 1).

Total area = 1: 2 a + 2 b = 1

E(X) = $\int x(a + bx)\,dx = 2a + \frac{8}{3}b = \frac{17}{15}$

Solving the simultaneous equations: a = $\frac{3}{10}$, b = $\frac{1}{5}$

P(X < 1) = $\int f(x)\,dx$ from 0 to 1 = $\frac{3}{10} \times 1 + \frac{1}{5} \times 0.5$

= $\frac{2}{5}$

Answer: (a) a = $\frac{3}{10}$, b = $\frac{1}{5}$ (b) $\frac{2}{5}$

7 Quadratic sequence from recurrence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = u_n + 2n + 2$, $u_1 = 6$.

(a) Show that $u_n = an^2 + bn + c$ and find a, b, c.

(b) Find the least n for which $u_n > 849$.

$$u_n = u_1 + \sum_{k=1}^{n-1}(2k + 2) = 6 + 2 \times \frac{(n-1)n}{2} + 2(n-1).$$

$= n^2 + n + 4$.

So a = 1, b = 1, c = 4.

$u_{28} = 816 \leq 849$ and $u_{29} = 874 > 849$.

Answer: (a) a = 1, b = 1, c = 4 (b) n = 29

8 Exponential times trigonometric (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = e^{-2x} \cos x$. Show that $dy/dx = e^{-2x}(-2\cos x - \sin x)$.

Find the x coordinate of the stationary point with the smallest positive x (3 s.f.), and its nature.

Product rule: $dy/dx = (-2)e^{-2x} \cos x - e^{-2x} \times \sin x$

$dy/dx = 0$ requires $-2 \cos x = 1 \sin x$

$\tan x = -2$, smallest positive x = 2.0344

x = 2.03

Second derivative test (or gradient change): a minimum

Answer: x = 2.03, minimum

9 Distance travelled with a change of direction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time t seconds its velocity is v m s^{-1}, where

$v = 3t^2 - 21t + 30$ for $0 \leq t \leq 6$

Find (a) the times at which P is at instantaneous rest, (b) the total distance travelled by P in the first 6 seconds.

$v = 3(t - 2)(t - 5)$, so v = 0 when t = 2 and t = 5

v > 0 for 0 < t < 2, v < 0 for 2 < t < 5, v > 0 for t > 5: direction changes at each

$s = \int v\,dt = \frac{3}{3}t^3 - 10.5t^2 + 30t$

Distances: 0 to 2: 26; 2 to 5: $\frac{27}{2}$; 5 to 6: $\frac{11}{2}$

Total = $26 + \frac{27}{2} + \frac{11}{2}$

Answer: (a) t = 2, t = 5 (b) 45 m

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10 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$\frac{dy}{dx} = \frac{3x^2}{y}$, $y > 0$, and $y = 4$ when $x = 0$.

Find y in terms of x, and the value of y when x = 4.

$\int y\,dy = \int 3x^2\,dx$ gives $\frac{y^2}{2} = 1x^3 + c$

$x = 0$, $y = 4$: $c = 8$, so $y^2 = 2x^3 + 16$

$x = 4$: $y^2 = 144$

Answer: $y = \sqrt{2x^3 + 16}$; $y = 12$