Day 147	A Level Maths: S Tier 10-a-day
1 A shell is projected from level ground with horizontal and vertical velocity components 19.6 and 44.1 m s^{-1}. Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points. Take g = 9.8 m s^{-2}.	
2 $f(x) = \frac{12}{x^2 + 2x + 3}$ for all real x. Find the greatest value of f(x), the value of x at which it occurs, and the range of f.	
3 The equation $x^3 = 4x + 16$ has a root α. (a) Show that the equation can be written as $x = \sqrt[3]{(4x + 16)}$. (b) Use $x_{n+1} = \sqrt[3]{(4x_n + 16)}$ with $x_0 = 3$ to find x_1, x_2, x_3 to 4 d.p., and hence state α to 3 d.p.	
4 Find the set of values of x for which both $x^2 - x - 20 \leq 0$ and $-2x - 5 < 3$.	
5 A ball is projected from the top of a vertical cliff, 19.6 m high, at 24.5 m s^{-1} at angle α above the horizontal ($\tan \alpha = \frac{3}{4}$), and lands on horizontal ground. Find (a) the time of flight and the distance from the cliff, (b) the speed and direction on landing. Take g = 9.8 m s^{-2}.	

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6 $P(X = x) = kx(6 - x)$ for $x = 1, 2, ..., 5$, and 0 otherwise.

(a) Show that $k = \frac{1}{35}$.

(b) Find E(X) and Var(X).

(c) Find Var(4X + 5).

7 The curve $y = 2x^2\sqrt{180 - x}$, for $0 < x < 180$, has one stationary point. Find its exact coordinates and determine its nature.

8 $X \sim N(\mu, 64)$. H_0: $\mu = 52$ is tested against H_1: $\mu > 52$ using the mean of a sample of size 25 at the 5% level.

(a) Find the critical region for the sample mean.

(b) Find the probability of a Type II error when $\mu = 54.5$.

9 Each year Jo pays £250 into an account on 1 January.
Interest of 3% is added each 31 December.
Find the first year-end at which the balance exceeds £15,000.

10 Two drones P and Q move so that, t seconds after launch, their position vectors in metres are

$$\mathbf{r}_P = \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} \text{ and } \mathbf{r}_Q = \begin{pmatrix} -7 \\ 5 \\ -1 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}.$$

Find the time at which they are closest and the exact minimum distance between them.