Day 143 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Sectors: perimeter and area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius r cm and angle θ radians, where $\theta < 2\pi$.
Its perimeter is 26 cm and its area is 42 cm^2.
Find the two possible pairs of values of r and θ.

Arc = rθ, so $2r + r\theta = 26$ and $\frac{1}{2}r^2\theta = 42$

$r\theta = 26 - 2r$, so $\frac{1}{2}r(26 - 2r) = 42$

$r^2 - 13r + 42 = 0$, so $(r - 6)(r - 7) = 0$

r = 6: rθ = 14, $\theta = \frac{7}{3}$ r = 7: rθ = 12, $\theta = \frac{12}{7}$

Answer: r = 6, $\theta = \frac{7}{3}$ or r = 7, $\theta = \frac{12}{7}$

2 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from level ground with horizontal and vertical velocity components 14.7 and 39.2 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take g = 9.8 m s^{-2}.

At 45° to the horizontal, $|v_y| = v_x = 14.7$
Moving upwards: $39.2 - 9.8t = 14.7$, so t = 2.5
Moving downwards: $39.2 - 9.8t = -14.7$, so t = 5.5
Horizontal distance = $14.7 \times (5.5 - 2.5)$
Answer: (a) 2.5 s and 5.5 s (b) 44.1 m

3 Trapezium rule and over/under estimates (AQA, Edexcel, OCR, MEI, Eduqas (trapezium rule))

Use the trapezium rule with 4 strips to estimate

$\int_0^2 \frac{1}{\sqrt{1+x}}\,dx$, to 4 s.f.

Is it an overestimate or an underestimate? Explain.

$h = \frac{1}{2}$; ordinates at x = 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, …, 2

Sum = $\frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)] = 1.472$

f''(x) > 0 on the interval, so the curve is convex and the trapezia lie above the curve
Answer: 1.472; overestimate

4 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The size of a rod is N(62, 7^2) mm. A rod is 'large' if it exceeds 67.
(a) Find the probability that a rod is large.
(b) 12 rods are chosen at random. Find the probability that at least 3 are large.

(a) P(X > 67) = P(Z > 0.714) = 0.2375
(b) Let Y = number large, Y ~ B(12, 0.2375)
$P(Y \geq 3) = 1 - P(Y \leq 2) = 0.570$
Answer: (a) 0.238 (b) 0.570

5 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes
(dashed) and a point P. C has equation

$y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes x = 1 and y = 2, so p = −1 and q = 2.

$y = \frac{a}{x - 1} + 2$.

P(−1, 1): $1 = \frac{a}{-2} + 2$, so a = 2.

Answer: a = 2, p = −1, q = 2

6 Variable force in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P of mass 4 kg moves under a single force F N, where $\mathbf{F} = (24t + 20)\mathbf{i} + (36t^2 - 8)\mathbf{j}$ and t is the time in seconds. At t = 0, P is at the origin with velocity $(3\mathbf{i})$ m s^{-1}.
Find (a) the velocity of P at t = 2, (b) the position vector of P at t = 2.

$\mathbf{a} = \frac{\mathbf{F}}{4} = (6t + 5)\mathbf{i} + (9t^2 - 2)\mathbf{j}$

$\mathbf{v} = \int \mathbf{a}\, dt = (3t^2 + 5t + 3)\mathbf{i} + (3t^3 - 2t)\mathbf{j}$ (using $\mathbf{v}(0) = (3\mathbf{i})$)
At t = 2: $\mathbf{v} = (25\mathbf{i} + 20\mathbf{j})$
$\mathbf{r} = \int \mathbf{v}\, dt = (t^3 + 2.5t^2 + 3t)\mathbf{i} + (0.75t^4 - t^2)\mathbf{j}$
At t = 2: $\mathbf{r} = (24\mathbf{i} + 8\mathbf{j})$
Answer: (a) $(25\mathbf{i} + 20\mathbf{j})$ m s^{-1} (b) $(24\mathbf{i} + 8\mathbf{j})$ m

7 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both
$x^2 + 2x - 8 \leq 0$ and $3x + 7 \geq 10$.

Quadratic: $(x + 4)(x - 2) \leq 0$ gives $-4 \leq x \leq 2$.
Linear: $3x + 7 \geq 10$ gives $x \geq 1$.
Both must hold: take the overlap of the two sets.
Answer: $1 \leq x \leq 2$

8 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(−4, −5) and B(−1, −4). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c - (-4))^2 + 25 = (c - (-1))^2 + 16$.
Expand and solve: $6c = -24$, so $c = -4$.
Area = $\frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$
$= \frac{1}{2}|0 \times (-4) - 3 \times (-5)| = \frac{15}{2}$.

Answer: C(−4, 0); area $\frac{15}{2}$

9 Integration by substitution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution u = 1 − x to find the exact value of

$\int_0^1 x(1 - x)^2\, dx$

x = 1 − u, dx = −du; limits become u = 1 and u = 0

Integral = $\int_0^1 (1 - u)u^2\, du = \int_0^1 (u^2 - u^3)\, du$

Evaluate the limits

Answer: $\frac{1}{12}$

10 Poisson: sum of two independent variables, conditional (CIE)

X and Y are independent, X ~ Po(4) and Y ~ Po(2).
(a) State the distribution of X + Y.
(b) Given that X + Y = 7, find the probability that X = 1.

X + Y ~ Po(6)

$P(X = 1 \mid X + Y = 7) = \frac{P(X = 1)P(Y = 6)}{P(X + Y = 7)}$

$= \frac{e^{-4}4^1}{1! \times e^{-2}2^6/6!} \div \frac{e^{-6}6^7}{7!}$

$= C(7, 1)\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^6$, a binomial probability

= 0.00640

Answer: (a) Po(6) (b) 0.00640