Day 136 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves with constant acceleration. At t = 0 seconds P is at $(4\mathbf{i} - 3\mathbf{j})$ m with velocity $(\mathbf{i} + 14\mathbf{j})$ m s^{-1}. At t = 6, P has velocity $(7\mathbf{i} + 2\mathbf{j})$ m s^{-1}.
Find (a) the acceleration of P, (b) the time at which P moves parallel to $(3\mathbf{i} + 2\mathbf{j})$, (c) the position vector of P at that time.

$a = \frac{v - u}{t} = \frac{(7\mathbf{i} + 2\mathbf{j}) - (\mathbf{i} + 14\mathbf{j})}{6} = (\mathbf{i} - 2\mathbf{j})$

$v = (\mathbf{i} + 14\mathbf{j}) + t(\mathbf{i} - 2\mathbf{j})$

Parallel to $(3\mathbf{i} + 2\mathbf{j})$: $(t + 1) \times 2 = (-2t + 14) \times 3$, giving t = 5

$r = (4\mathbf{i} - 3\mathbf{j}) + 5(\mathbf{i} + 14\mathbf{j}) + \frac{1}{2} \times 5^2 (\mathbf{i} - 2\mathbf{j}) = (21.5\mathbf{i} + 42\mathbf{j})$

Answer: (a) $(\mathbf{i} - 2\mathbf{j})$ m s^{-2} (b) t = 5 (c) $(21.5\mathbf{i} + 42\mathbf{j})$ m

2 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A spinner is said to land on blue with probability 0.2. In 30 trials it does so 3 times.
Test at the 10% level whether the probability is different from 0.2.

H_0: p = 0.2, H_1: $p \neq 0.2$ (two-tailed). X ~ B(30, 0.2)

3 is below the mean 6, so use the lower tail: $P(X \leq 3) = 0.123$

Each tail is compared with 5%: 0.123 is greater than 5%

Do not reject H_0: there is insufficient evidence that the probability is different from 0.2

Answer: Do not reject H_0 (p-value 0.123 against 5%)

3 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ax^n$ passes through (1, 3) and (3, 27).
Find the values of a and n.

$3 = a \times 1^n$ and $27 = a \times 3^n$

Divide: $3^n = 9$ so n = 2

$a = 3 \div 1^2 = 3$

Answer: a = 3, n = 2

4 Cubic graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the cubic y = f(x), which touches the x-axis once. Find f(x), giving your answer in expanded form.

Crosses at x = −1 and touches at x = 4, so $f(x) = k(x + 1)(x - 4)^2$.

At (0, 32): $32 = k(1)(-4)^2 = 16k$, so k = 2.

$f(x) = 2(x + 1)(x - 4)^2$, then expand.

Answer: $f(x) = 2x^3 - 14x^2 + 16x + 32$

5 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2y + xy^2 = 84$.
Find the equation of the tangent to C at the point (4, 3),
in the form ax + by + c = 0 with integers a, b, c.

Differentiate implicitly with respect to x

At (4, 3): $dy/dx = -\frac{33}{40}$

$y - 3 = (-\frac{33}{40})(x - 4)$, then clear fractions

Answer: $33x + 40y - 252 = 0$

6 Proof by contradiction (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove by contradiction that $\log_9 6$ is irrational.

Assume $\log_9 6 = \frac{m}{n}$, m, n positive integers.

Then $9^{\frac{m}{n}} = 6$, so $9^m = 6^n$.
Prime factors of 9^m and 6^n must be identical,
but 2 divides 6^n but 2 does not divide 9^m.
This is a contradiction, so $\log_9 6$ is irrational.
Answer: Proved

7 Stationary points: exact coordinates (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the stationary point of
$y = 3x \ln x$ for $x > 0$,
and determine its nature.

$dy/dx = 3(\ln x + 1) = 0$ gives $\ln x = -\frac{1}{1}$

$y = -3e^{-1}$ when $x = e^{-1}$
Second derivative at this point is positive, so it is a minimum
Answer: $(e^{-1}, -3e^{-1})$, minimum

8 Force up a rough slope: find weight and friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of weight W N is on a rough plane inclined at α to the horizontal, where $\sin\alpha = \frac{5}{13}$. A force P N acts up a line of greatest slope; the particle stays at rest for $2.2 \leq P \leq 107.8$.
Find (a) W, (b) the coefficient of friction, (c) the acceleration when P = 215.6. Take g = 9.8 m s^{-2}.

Limiting friction acts down the plane at P = 107.8 and up the plane at P = 2.2
$P_{max} = W\sin\alpha + F$ and $P_{min} = W\sin\alpha - F$, where $F = \mu W\cos\alpha$

Adding: $2W\sin\alpha = 110$, so $W = \frac{110}{2 \times \frac{5}{13}} = 143$

Subtracting: $2F = 105.6$, so $F = 52.8 = \mu \times 143 \times \frac{12}{13}$, giving $\mu = 0.4$

P = 215.6: net force up the plane = 215.6 − 107.8 = 107.8, and mass = $\frac{143}{9.8}$

Answer: (a) 143 N (b) 0.4 (c) 7.39 m s^{-2}

9 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(6, −3) and B(2, −5). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c-6)^2 + 9 = (c-2)^2 + 25$.
Expand and solve: $-8c = -16$, so c = 2.
Area = $\frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$
$= \frac{1}{2}|4 \times (-5) - 0 \times (-3)| = 10$.
Answer: C(2, 0); area 10

10 Normal test: critical region and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(\mu, 100)$. H_0: $\mu = 40$ is tested against H_1: $\mu < 40$ using the mean of a sample of size 16 at the 10% level.

(a) Find the critical region for the sample mean.

(b) Find the probability of a Type II error when $\mu = 35$.

Under H_0, sample mean ~ $N(40, \frac{100}{16})$, standard error = 2.500

z = 1.2816 for the lower tail: critical value = $40 - 1.2816 \times 2.500 = 36.80$

Type II error: do not reject H_0, sample mean > 36.80, where mean ~ N(35, 2.500)

$z = \frac{36.80 - 35}{2.500} = 0.718$

Probability = 0.236

Answer: (a) sample mean < 36.8 (b) 0.236