Day 135 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Combined mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Class P has 30 students with mean mark 59 and standard deviation 6. Class Q has 20 students with mean mark 80 and standard deviation 4.
(a) Find the mean mark of all 50 students.
(b) Find the standard deviation of all 50 students.

Σx for P = 30 × 59 = 1770, Σx for Q = 20 × 80 = 1600

Combined mean = $\frac{3370}{50}$ = 67.4

$\Sigma x^2 = n(\sigma^2 + \text{mean}^2)$: 30(36 + 3481) + 20(16 + 6400) = 233830

Variance = $\frac{233830}{50} - 67.400^2$ = 133.84, so σ = 11.57

Answer: (a) 67.4 (b) 11.6

2 Force applied then removed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 15 kg is at rest on rough horizontal ground (μ = 0.1). A horizontal force of 35.7 N acts on it for 7 s and is then removed.
Find (a) the speed when the force is removed, (b) the total distance travelled before the box stops. Take g = 9.8 m s^{-2}.

Friction = μmg = 0.1 × 15 × 9.8 = 14.7 N
While pushed: 35.7 − 14.7 = 15 a, so a = 1.4
v = 1.4 × 7 = 9.8; s_1 = ½ × 1.4 × 7^2 = 34.3

After release: deceleration = μg = 0.98, so s_2 = $\frac{9.8^2}{2 \times 0.98}$ = 49

Total distance = 34.3 + 49

Answer: (a) 9.8 m s^{-1} (b) 83.3 m

3 Logarithmic equations with ln (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve 2 ln x = ln(x + 60) + ln 2,
stating clearly why one root is rejected.

ln x^2 = ln(2(x + 60)), so x^2 = 2x + 120
$x^2 - 2x - 120 = 0$, $(x - 12)(x + 10) = 0$
x = −10 is rejected since ln x is undefined for x < 0
Answer: x = 12

4 Remainder theorem (quadratic divisor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

When a polynomial f(x) is divided by (x − 3)
the remainder is −19; when divided by (x + 2)
the remainder is 1. Find the remainder when
f(x) is divided by (x − 3)(x + 2).

f(x) = (x − 3)(x + 2)Q(x) + px + q.
f(3) = −19: 3p + q = −19.
f(−2) = 1: −2p + q = 1.
Subtracting: 5p = −20, so p = −4, q = −7.
Answer: −4x − 7

5 Rejected batches: binomial inside binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box holds 9 components, each faulty with probability 0.1, independently. A box is rejected if it has at least 2 faulty components.
(a) Find the probability that a box is rejected.
(b) 14 boxes are inspected. Find the probability that at least 2 are rejected.

Let X ~ B(9, 0.1). P(rejected) = P(X ≥ 2) = 1 − P(X ≤ 1) = 0.2252
Let Y = number of rejected boxes, Y ~ B(14, 0.2252)
P(Y ≥ 2) = 1 − P(Y ≤ 1) = 0.857
Answer: (a) 0.225 (b) 0.857

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6 Surds: rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Show that $\frac{3+\sqrt{7}}{3-\sqrt{7}} - \frac{3-\sqrt{7}}{3+\sqrt{7}}$ can be written in the form $k\sqrt{7}$, and find k.

Common denominator $(3-\sqrt{7})(3+\sqrt{7}) = 9 - 7 = 2$.
Numerator: $(3+\sqrt{7})^2 - (3-\sqrt{7})^2$
$= (9 + 7 + 23\sqrt{7}) - (9 + 7 - 23\sqrt{7}) = 12\sqrt{7}$.
So the expression $= \frac{12\sqrt{7}}{2}$.

Answer: k = 6

7 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2$, $y = t^3 + 9t$.
Find the coordinates of the points on the curve where the gradient is -3.

$dx/dt = 4t$, $dy/dt = 3t^2 + 9$, so $dy/dx = \frac{3t^2+9}{4t}$
Set equal to -3: $3t^2 + 12t + 9 = 0$
$t = -3$ or $t = -1$
Answer: (18, −54) and (2, −10)

8 Proof by contradiction (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove by contradiction that $\log_5 15$ is irrational.

Assume $\log_5 15 = \frac{m}{n}$, m, n positive integers.

Then $5^{\frac{m}{n}} = 15$, so $5^m = 15^n$.
Prime factors of 5^m and 15^n must be identical,
but 3 divides 15^n but 3 does not divide 5^m.
This is a contradiction, so $\log_5 15$ is irrational.
Answer: Proved

9 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$\frac{dy}{dx} = 6xy$, and $y = 5$ when $x = 0$.
Solve the differential equation and find y when $x = 2$.

$\int \frac{1}{y}\,dy = \int 6x\,dx$, so $\ln y = 6x^2/2 + c = 3x^2 + c$
$x = 0$, $y = 5$: $c = \ln 5$, so $y = 5\,e^{3x^2}$
$x = 2$: exponent = 12
Answer: $y = 5\,e^{3x^2}$; $y = 5e^{12}$

10 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (1 kg) lies on a rough table with $\mu = 0.3$, and is joined by a light string over a smooth pulley (see diagram) to particle B (1 kg) on a smooth slope, $\alpha = 30°$.
Released from rest, the string is cut when B has moved 3 m. Find the further distance A slides before stopping.

A: $T - 0.3 \times 1 \times 9.8 = 1\,a$. B: $1 \times 9.8 \times \frac{1}{2} - T = 1\,a$

Adding: $9.8(1 \times \frac{1}{2} - 0.3 \times 1) = 2\,a$, so $a = 0.98$

$T = 1(0.98 + 0.3 \times 9.8) = 3.92$

When cut: $v^2 = 2 \times 0.98 \times 3 = 5.88$. Then A decelerates at $\mu g = 2.94$

Further distance $= \frac{5.88}{2 \times 2.94} = 1$

Answer: 1 m