Day 126 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Normal approximation to a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(80, 0.2)$.
(a) State why a normal approximation is suitable for X.
(b) Use a normal approximation to estimate $P(9 \leq X \leq 19)$, to 3 significant figures.

np = 16 and nq = 64, both greater than 5 (n large, p not extreme)
$Y \sim N(16, 12.8)$ with $\sigma = 3.578$
Apply the continuity correction:
$P(9 \leq X \leq 19) \approx P(8.5 < Y < 19.5) = 0.8360 - 0.01803 = 0.818$
Answer: 0.818

2 Equations in disguise (substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 1\sqrt{x} - 20 = 0$.

Let $u = \sqrt{x}$: $u^2 - 1u - 20 = 0$.
$(u - 5)(u + 4) = 0$, so $\sqrt{x} = 5$ or $\sqrt{x} = -4$.
$\sqrt{x}$ cannot be negative, so reject $\sqrt{x} = -4$.
$x = 25$.
Answer: x = 25

3 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cot(\theta + 30°) = 1$
for $-90° \leq \theta \leq 270°$.

Let $u = \theta + 30°$; then $\cot u = 1$
Base solutions: $u = 45°$ (+ multiples of 180°)
$\theta = (u - 30°) \div 1$, keeping those in range
Answer: θ = 15°, 195°

4 Symmetric limits of a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Scores are $N(35, 36^2)$. The highest 10% and the lowest 10% of scores are discarded.
Find the range of scores that remain, correct to 3 significant figures.

The remaining middle 80% lies between $z = \pm 1.2816$
Lower limit = $35 - 1.2816 \times 6 = 27.31$
Upper limit = $35 + 1.2816 \times 6 = 42.69$
Answer: 27.3 to 42.7

5 Addition formulae with exact values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is acute and B is acute. $\sin A = \frac{7}{25}$ and $\cos B = \frac{15}{17}$.
Find the exact values of $\sin(A + B)$ and $\tan(A - B)$.

$\cos A = \frac{24}{25}$, $\sin A = \frac{7}{25}$; $\sin B = \frac{8}{17}$, $\cos B = \frac{15}{17}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{297}{425}$

$\tan(A - B) = \frac{\sin(A - B)}{\cos(A - B)} = \frac{-\frac{87}{425}}{\frac{416}{425}} = -\frac{87}{416}$

Answer: $\sin(A + B) = \frac{297}{425}$, $\tan(A - B) = -\frac{87}{416}$

6 Partial fractions (repeated factor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-18x^2 + 21x - 15}{(x-2)(2x-1)^2}$ in the form $\frac{A}{(x-2)} + \frac{B}{(2x-1)} + \frac{C}{(2x-1)^2}$.

$-18x^2 + 21x - 15 \equiv A(2x-1)^2 + B(x-2)(2x-1) + C(x-2)$.

Put $x = 2$: $-45 = A(3)^2$, so $A = -5$.

Put $x = \frac{1}{2}$: $-9 = C(-\frac{3}{2})$, so $C = 6$.

Compare x^2: $-18 = 4A + 2B$, so $B = 1$.

Answer: $A = -5$, $B = 1$, $C = 6$

7 Two particles over a pulley: height reached (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 12 kg and 6 kg hang on a light string over a smooth fixed pulley. The 12 kg particle is 0.75 m above the ground; the 6 kg one is on the ground.

Released from rest, find (a) the tension, (b) the greatest height reached by the 6 kg particle. Take $g = 9.8$ m s^{-2}.

$12g - T = 12a$ and $T - 6g = 6a$, so $a = \frac{12-6}{12+6} \times 9.8 = \frac{49}{15}$

$T = 12(9.8 - \frac{49}{15}) = 78.4$

The 6 kg particle rises 0.75 m before the 12 kg particle lands, reaching $v^2 = 2 \times \frac{49}{15} \times 0.75 = 4.9$

The string goes slack, so it then moves freely under gravity: extra height $= \frac{v^2}{2g} = \frac{4.9}{19.6} = 0.25$

Greatest height $= 0.75 + 0.25$

Answer: (a) 78.4 N (b) 1 m

8 Car on a hill: power and maximum speed (CIE)

A car of mass 1200 kg, with engine power 6.704 kW, climbs a straight road with $\sin\alpha = \frac{1}{25}$ against a constant resistance of 200 N.

Find (a) the greatest steady speed up the hill, (b) the acceleration at $\frac{10}{3}$ m s^{-1}. Take $g = 9.8$ m s^{-2}.

Steady speed: driving force $= 200 + 1200 \times 9.8 \times \frac{1}{25} = 670.4$ N

$P = Fv$, so $v = \frac{6704}{670.4} = 10$

At $\frac{10}{3}$ m s^{-1}: $F = \frac{6704}{\frac{10}{3}} = 2011.2$ N

$F - 670.4 = 1200a$, so $a = \frac{1340.8}{1200}$

Answer: (a) 10 m s^{-1} (b) 1.12 m s^{-2}

9 Indices: simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously, giving integer x and y:

$8^x \times 2^y = 16$

$27^x \div 3^y = 9$

Write each side with prime bases: $16 = 2^4$, $9 = 3^2$.

First: $2^{3x} \times 2^{1y} = 2^4$, so $3x + 1y = 4$.

Second: $3^{3x-1y} = 3^2$, so $3x - 1y = 2$.

Solving the pair of linear equations: $x = 1$, $y = 1$.

Answer: $x = 1$, $y = 1$

10 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{1}{12 + 4\sin\theta + 3\cos\theta}$

(a) Write $4\sin\theta + 3\cos\theta$ as $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.

(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{4^2 + 3^2} = 5$, $\tan\alpha = \frac{3}{4}$, $\alpha = 36.9°$

Denominator $= 12 + 5\sin(\theta + 36.9°)$, which lies between 7 and 17

Greatest $f(\theta) = \frac{1}{7} = \frac{1}{7}$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{1}{17} = \frac{1}{17}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 36.9° = 270°$, so $\theta = 233.1°$

Answer: (a) $5\sin(\theta + 36.9°)$ (b) greatest $\frac{1}{7}$, least $\frac{1}{17}$, $\theta = 233.1°$