Day 12 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 30 and upper quartile 43. An outlier is a value more than 2 × IQR beyond the nearest quartile.

(a) Find the range of values that are not outliers.

(b) 8 observations have mean 38. The largest is 79, an outlier, and it is removed. Find the new mean.

IQR = 43 − 30 = 13, so 2 × IQR = 26

Lower limit = 30 − 26 = 4, upper limit = 43 + 26 = 69

$\Sigma x = 8 \times 38 = 304$, so without 79: $\Sigma x = 225$

New mean = $\frac{225}{7}$

Answer: (a) 4 to 69 (b) 32.14

2 Particle on a smooth slope joined to hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (7 kg) and B (8 kg) are joined by a light string over a smooth pulley (see diagram). A lies on a smooth slope, $\alpha = 30°$, and B is 1.25 m above the ground.

Released from rest, find the further distance A moves up the slope after B lands (A does not reach the pulley).

A: $T - 7 \times 9.8 \times \frac{1}{2} = 7a$. B: $8 \times 9.8 - T = 8a$

Adding: $9.8(8 - 7 \times \frac{1}{2}) = 15a$, so $a = 2.94$

$T = 8(9.8 - 2.94) = 54.88$

When B lands: $v^2 = 2 \times 2.94 \times 1.25 = 7.35$

Then A decelerates at $g \sin \alpha = 4.9$: further distance = $\frac{7.35}{2 \times 4.9} = 0.75$

Answer: 0.75 m

3 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows y = f(x), made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of

$y = -f(\frac{1}{2}x - 1)$.

Inside f: replace x by $\frac{1}{2}x - 1$, so x-coordinates become $(x + 1) \div \frac{1}{2}$.

Outside: y-coordinates become $-1 \times y$.

A: (−6, −1) B: (2, 2)

C: (6, −5) D: (10, 4)

Answer: A′(−6, −1), B′(2, 2), C′(6, −5), D′(10, 4)

4 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas, CIE (small angle))

θ is small and measured in radians. Use small angle approximations to find the value that

$\frac{\sin 1\theta \tan 3\theta}{1 - \cos 2\theta}$

approaches as $\theta \to 0$.

$\sin 1\theta \approx 1\theta$, $\tan 3\theta \approx 3\theta$, $1 - \cos 2\theta \approx 2\theta^2$

Expression $\approx \frac{1\theta \times 3\theta}{2\theta^2} = \frac{3}{2}$

Answer: $\frac{3}{2}$

5 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x^3 - 6x - 4 \leq 0$.

Use the factor theorem to find roots: −1, 2.

$f(x) = 2(x + 1)^2(x - 2)$.

Sign of f in each region (left to right): −, −, +.

Take the regions where $f \leq 0$ (include roots if ≤ or ≥).

Answer: $x \leq 2$

6 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 3x^3\sqrt{112 - x}$, for $0 < x < 112$, has one stationary point. Find its exact coordinates and determine its nature.

$\frac{dy}{dx} = \frac{3x^2(672 - 7x)}{2\sqrt{112 - x}}$

$dy/dx = 0$: $672 - 7x = 0$, so $x = 96$

$y = 3 \times 96^3 \times 4$

Gradient changes from positive to negative: a maximum

Answer: (96, 10616832), maximum

7 Partial fractions (improper) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{2x^3 - 16x^2 + 40x - 20}{(x - 4)(x - 2)}$ in the form $ax + b + \frac{C}{(x - 4)} + \frac{D}{(x - 2)}$.

Divide: the numerator has degree 3, the denominator degree 2.

$(2x^3 - 16x^2 + 40x - 20) \div (x^2 - 6x + 8)$ gives quotient $2x - 4$.

Remainder: 12.

Put $x = 4$: remainder = $C(2)$ so $C = 6$. Put $x = 2$: $D = -6$.

Answer: $2x - 4 + \frac{6}{(x - 4)} - \frac{6}{(x - 2)}$

8 Points of inflection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the point of inflection of $y = (x - 2)e^{-x}$, justifying that it is a point of inflection.

$dy/dx = e^{-x}(1 - (x - 2))$

$d^2y/dx^2 = e^{-x}((x - 2) - 2) = 0$, so $x = 4$

d^2y/dx^2 changes sign either side of $x = 4$

$y = 2e^{-4}$

Answer: $(4, 2e^{-4})$

9 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The fill volume of a bottle is N(μ, 4) with known variance 4. The mean was claimed to be 30 ml. A random sample of 36 has mean 29.25. Test at the 10% level whether the mean is greater than 30.

H_0: $\mu = 30$, H_1: $\mu > 30$. Under H_0, the sample mean ~ $N(30, \frac{4}{36})$

$z = \frac{29.25 - 30}{2 \div \sqrt{36}} = -2.250$

p-value = $P(Z > -2.250) = 0.988$

$0.988 > 10\%$, so do not reject H_0

There is insufficient evidence at the 10% level that the mean has changed as claimed

Answer: Do not reject H_0 ($z = -2.250$, p-value 0.988)

10 Variable force in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P of mass 2 kg moves under a single force F N, where $\mathbf{F} = (12t + 8)\mathbf{i} + (12t^2 - 10)\mathbf{j}$ and t is the time in seconds. At $t = 0$, P is at the origin with velocity $(4\mathbf{i} - \mathbf{j})$ m s^{-1}. Find (a) the velocity of P at $t = 2$, (b) the position vector of P at $t = 2$.

$\mathbf{a} = \frac{\mathbf{F}}{2} = (6t + 4)\mathbf{i} + (6t^2 - 5)\mathbf{j}$

$\mathbf{v} = \int \mathbf{a}\, dt = (3t^2 + 4t + 4)\mathbf{i} + (2t^3 - 5t - 1)\mathbf{j}$ (using $\mathbf{v}(0) = (4\mathbf{i} - \mathbf{j})$)

At $t = 2$: $\mathbf{v} = (24\mathbf{i} + 5\mathbf{j})$

$\mathbf{r} = \int \mathbf{v}\, dt = (t^3 + 2t^2 + 4t)\mathbf{i} + (0.5t^4 - 2.5t^2 - t)\mathbf{j}$

At $t = 2$: $\mathbf{r} = (24\mathbf{i} - 4\mathbf{j})$

Answer: (a) $(24\mathbf{i} + 5\mathbf{j})$ m s^{-1} (b) $(24\mathbf{i} - 4\mathbf{j})$ m