Day 114 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (1 kg) lies on a rough table with $\mu = 0.4$, and is joined by a light string over a smooth pulley (see diagram) to particle B (9 kg) on a smooth slope, $\sin\alpha = \frac{3}{5}$.

Released from rest, the string is cut when B has moved 1 m. Find the further distance A slides before stopping.

A: $T - 0.4 \times 1 \times 9.8 = 1\,a$. B: $9 \times 9.8 \times \frac{3}{5} - T = 9\,a$

Adding: $9.8(9 \times \frac{3}{5} - 0.4 \times 1) = 10\,a$, so $a = 4.9$

$T = 1(4.9 + 0.4 \times 9.8) = 8.82$

When cut: $v^2 = 2 \times 4.9 \times 1 = 9.8$. Then A decelerates at $\mu g = 3.92$

Further distance $= \frac{9.8}{2 \times 3.92} = 1.25$

Answer: 1.25 m

2 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^3 + xy^2 = 54$.
Find the equation of the tangent to C at the point (3, 3),
in the form $ax + by + c = 0$ with integers a, b, c.

Differentiate implicitly with respect to x

At (3, 3): $dy/dx = -2$

$y - 3 = (-2)(x - 3)$, then clear fractions

Answer: $2x + y - 9 = 0$

3 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

15 recorded masses (grams) have mean 58 and standard deviation 6.
One value was recorded as 45 but should have been 59.
Find the correct mean and the correct standard deviation.

$\Sigma x = 15 \times 58 = 870$, so corrected $\Sigma x = 870 - 45 + 59 = 884$

Correct mean $= \frac{884}{15} = 58.93$

$\Sigma x^2 = 15(36 + 3364) = 51000$, corrected $\Sigma x^2 = 51000 - 2025 + 3481 = 52456$

Variance $= \frac{52456}{15} - 58.933^2 = 23.929$, $\sigma = 4.892$

Answer: mean 58.9, standard deviation 4.89

4 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes
(dashed) and a point P. C has equation

$y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes $x = -1$ and $y = -2$, so $p = 1$ and $q = -2$.

$y = \frac{a}{x + 1} - 2$.

P(1, −3): $-3 = \frac{a}{2} - 2$, so $a = -2$.

Answer: $a = -2$, $p = 1$, $q = -2$

5 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of

$\sin(2\cos^{-1}(\frac{5}{13}))$.

Let $A = \cos^{-1}(\frac{5}{13})$, so $\cos A = \frac{5}{13}$ and $\sin A = \frac{12}{13}$

$\sin 2A = 2 \sin A \cos A = 2 \times \frac{12}{13} \times \frac{5}{13}$

Answer: $\frac{120}{169}$

6 Distance travelled with a change of direction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time t seconds its velocity is v m s^{-1}, where
$v = t^2 - 10t + 21$ for $0 \leq t \leq 9$
Find (a) the times at which P is at instantaneous rest, (b) the total distance travelled by P in the first 9 seconds.

$v = (t - 3)(t - 7)$, so $v = 0$ when $t = 3$ and $t = 7$
$v > 0$ for $0 < t < 3$, $v < 0$ for $3 < t < 7$, $v > 0$ for $t > 7$: direction changes at each

$s = \int v\,dt = \frac{1}{3}t^3 - 5t^2 + 21t$

Distances: 0 to 3: 27; 3 to 7: $\frac{32}{3}$; 7 to 9: $\frac{32}{3}$

Total = $27 + \frac{32}{3} + \frac{32}{3}$

Answer: (a) t = 3, t = 7 (b) $\frac{145}{3}$ m

7 Quadratic sequence from recurrence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = u_n + 4n + 1$, $u_1 = 7$.
(a) Show that $u_n = an^2 + bn + c$ and find a, b, c.
(b) Find the least n for which $u_n > 925$.

$u_n = u_1 + \sum_{k=1}^{n-1}(4k + 1) = 7 + 4 \times \frac{(n - 1)n}{2} + 1(n - 1)$.

$= 2n^2 - n + 6$.
So $a = 2$, $b = -1$, $c = 6$.
$u_{21} = 867 \leq 925$ and $u_{22} = 952 > 925$.
Answer: (a) a = 2, b = −1, c = 6 (b) n = 22

8 Simultaneous equations (linear and quadratic) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously:
$x + 2y = -6$
$x^2 + xy - y^2 = 36$

From the first equation $x = -6 - 2y$.
Substitute into the second equation and collect terms:
this gives $y^2 + 18y = 0$.
$y = -18$ or $y = 0$, giving $x = 30$ or $x = -6$.
Answer: (x, y) = (30, −18) and (−6, 0)

9 Partial fractions then integrate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{5x + 25}{(x + 4)^2}$ in the form $\frac{A}{x + 4} + \frac{B}{(x + 4)^2}$.
(b) Find the exact value of $\int_0^3 \frac{5x + 25}{(x + 4)^2}\,dx$.

$5x + 25 \equiv A(x + 4) + B$, so $A = 5$, and $B = 5$

Integral = $[5\ln(x + 4) - \frac{5}{x + 4}]$

Substitute limits 0 and 3

Answer: (a) A = 5, B = 5; (b) $\frac{15}{28} - 10\ln 2 + 5\ln 7$

10 Critical region, significance level and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A test of H_0: p = 0.1 against H_1: p > 0.1 uses a sample of 25 at the 5% level.

(a) Find the critical region for X, the number of successes.

(b) Find the actual significance level.

(c) Find P(Type II error) when in fact p = 0.3.

X ~ B(25, 0.1). P(X ≥ 6) = 0.0334 and P(X ≥ 5) = 0.0980

The first is at most 0.05 and the second is not, so the critical region is X ≥ 6

Actual significance level = P(X ≥ 6) = 0.0334

Type II error: H_0 is not rejected, so X ≤ 5, with X ~ B(25, 0.3)

P(X ≤ 5) = 0.193

Answer: (a) X ≥ 6 (b) 0.0334 (c) 0.193