Day 109 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Binomial: mean and variance given (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A binomial random variable X has mean 31.5 and variance 7.875.
(a) Find n and p.
(b) Find P(X = 34), correct to 3 significant figures.

$np = 31.5$ and $np(1 - p) = 7.875$

Divide: $1 - p = 7.875 \div 31.5 = \frac{1}{4}$, so $p = \frac{3}{4}$

$n = 31.5 \div \frac{3}{4} = 42$

$P(X = 34) = C(42, 34) (\frac{3}{4})^{34} (\frac{1}{4})^{8} = 0.102$

Answer: (a) n = 42, p = $\frac{3}{4}$ (b) 0.102

2 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has its centre in the first quadrant and touches both the x-axis and the y-axis. It passes through the point (4, 2). Find the two possible equations of the circle.

Centre (r, r), radius r: $(x - r)^2 + (y - r)^2 = r^2$.
Substitute (4, 2): $(4 - r)^2 + (2 - r)^2 = r^2$.
$r^2 - 12r + 20 = 0$.
$(r - 2)(r - 10) = 0$, so r = 2 or r = 10.
Answer: $(x - 2)^2 + (y - 2)^2 = 4$ or $(x - 10)^2 + (y - 10)^2 = 100$

3 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A shell is projected from a point O on horizontal ground with speed 49 m s^{-1} at an angle θ above the horizontal, where $\tan \theta = \frac{3}{4}$. Find (a) the time of flight, (b) the range, (c) the greatest height. Take g = 9.8 m s^{-2}.

Components: horizontal 39.2, vertical 29.4 (as $\sin \theta = \frac{3}{5}$, $\cos \theta = \frac{4}{5}$)

Time of flight: $0 = 29.4 t - 4.9t^2$, so $t = \frac{29.4}{4.9} = 6$

Range = $39.2 \times 6 = 235.2$

Greatest height: $0 = 29.4^2 - 2 \times 9.8 \times H$, so $H = \frac{864.36}{19.6} = 44.1$

Answer: (a) 6 s (b) 235.2 m (c) 44.1 m

4 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of

$\tan(\tan^{-1}(\frac{4}{7}) + \tan^{-1}(1))$.

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{4}{7} + 1}{1 - \frac{4}{7} \times 1} = \frac{\frac{11}{7}}{\frac{3}{7}}$

Answer: $\frac{11}{3}$

5 Three-stage journey (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train accelerates uniformly from rest at 0.8 m s^{-2} to a speed of V m s^{-1}, travels at this speed, then decelerates uniformly at 1.5 m s^{-2} to rest.
The journey takes 68 s and covers 678 m. Find V and the time spent at constant speed.

Time accelerating = $\frac{V}{0.8}$, time decelerating = $\frac{V}{1.5}$, so time at constant speed = $68 - \frac{V}{0.8} - \frac{V}{1.5}$

Area under the speed-time graph: $678 = 68V - \frac{V^2}{1.6} - \frac{V^2}{3}$

Multiply by 2ab: $2.3V^2 - 163.2V + 1627.2 = 0$

$V = 12$ or $V = \frac{1356}{23}$

$V = \frac{1356}{23}$ gives time at constant speed -45 s, which is negative, so $V = 12$

Time at constant speed = $68 - 15 - 8 = 45$ s

Answer: V = 12, 45 s

6 Transformations of y = x^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe a sequence of two transformations that maps the curve $y = x^2$ onto the curve $y = 4x^2 + 16x + 13$.

Complete the square: $y = 4(x + 2)^2 - 3$.
Step 1: $y = x^2 \rightarrow y = 4x^2$: a stretch, scale factor 4, parallel to the y-axis.
Step 2: replace x by (x + 2) and add -3: translation by $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$.

Answer: a stretch, scale factor 4, parallel to the y-axis, then translation $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$

7 Stationary points of trigonometric curves (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coordinates of the stationary points of
$y = 2\cos 2x + 4\cos x$ for $0 < x < 2\pi$,
and determine their nature.

$\frac{dy}{dx} = -4\sin 2x - 4\sin x = -4\sin x(2\cos x + 1)$

$\frac{dy}{dx} = 0$: $\sin x = 0$ or $\cos x = -\frac{1}{2}$

$x = \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$ in the range

Test the sign of $\frac{d^2y}{dx^2}$ at each

Answer: $(\frac{2\pi}{3}, -3)$ minimum; $(\pi, -2)$ maximum; $(\frac{4\pi}{3}, -3)$ minimum

8 Poisson approximation to a binomial (CIE)

X ~ B(200, 0.01).
(a) Explain why a Poisson approximation is suitable and state its distribution.
(b) Use it to estimate P(X > 3), to 4 s.f.
(c) Find the exact value of P(X > 3), to 4 s.f.

n = 200 is large and p = 0.01 is small, with np = 2
X is approximately Po(2)
$P(X > 3) \approx 1 - P(Y \leq 3) = 0.1429$
Exact: $1 - P(X \leq 3)$ with X ~ B(200, 0.01) = 0.1420
Answer: (a) Po(2) (b) 0.1429 (c) 0.1420

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9 Range and domain by completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \dfrac{8}{x^2 + 2x + 5}$ for all real x.

Find the greatest value of f(x), the value of x at which it occurs, and the range of f.

$x^2 + 2x + 5 = (x + 1)^2 + 4$.

$(x + 1)^2 \geq 0$, so the denominator is at least 4, with equality at $x = -1$.

The denominator is always positive, so $f(x) > 0$.

Greatest $f = \dfrac{8}{4} = 2$ at $x = -1$.

Answer: greatest value 2 at $x = -1$; range $0 < f(x) \leq 2$

10 Perpendicular from a point to a line (3D) (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

The point P has coordinates (6, 6, −2).

Find the coordinates of the foot F of the perpendicular from P to l, and the exact distance PF.

F = (a + td) for some t, so $\mathrm{FP} = \begin{pmatrix} 0 \\ 5 \\ -3 \end{pmatrix} - t\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

FP · d = 0: −14 − t(14) = 0, so t = −1

F = (4, 2, −2)

$\mathrm{PF} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$, so $\mathrm{PF} = \sqrt{20}$

Answer: F = (4, 2, −2), PF = $2\sqrt{5}$