Day 108 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Double angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3\cos 2\theta - 9\sin\theta - 6 = 0$
for $0 \leq \theta < 360°$.

Use $\cos 2\theta \equiv 1 - 2\sin^2\theta$: $6\sin^2\theta + 9\sin\theta + 3 = 0$
$3(2\sin\theta + 1)(\sin\theta + 1) = 0$

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = -1$

Using the CAST diagram for $0 \leq \theta < 360°$
Answer: θ = 210°, 270°, 330°

2 Stone thrown from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 13 m s^{-1} from the top of a cliff. The top of the cliff is 26.4 m above the sea.
Find (a) the time taken for the stone to reach the sea, (b) the total distance travelled by the stone, to 3 s.f. Take g = 9.8 m s^{-2}.

Take upwards as positive: $-26.4 = 13t - 4.9t^2$, so $4.9t^2 - 13t - 26.4 = 0$

$t = \frac{13 + \sqrt{(13^2 + 19.6 \times 26.4)}}{9.8} = \frac{13 + 26.2}{9.8} = 4$

Height risen above the cliff top = $\frac{13^2}{(2 \times 9.8)} = 8.62$ m

Total distance = $2 \times 8.62 + 26.4$
Answer: (a) 4 s (b) 43.6 m

3 Binomial: finding p from a ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(5, p)$. It is given that $P(X = 2) = \frac{2}{5} \times P(X = 1)$.

(a) Find the value of p.
(b) Find $P(X = 0)$, correct to 3 significant figures.

$\frac{20}{2} p^2(1-p)^3 = \frac{2}{5} \times 5\, p(1-p)^4$

Divide by $p(1-p)^3$: $2p = \frac{2}{5}(1-p)$

$(2 + \frac{2}{5})p = \frac{2}{5}$

$p = \frac{1}{6}$

$P(X = 0) = (1-p)^5 = (\frac{5}{6})^5 = 0.402$

Answer: (a) $p = \frac{1}{6}$ (b) 0.402

4 Proof by deduction (divisibility) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $n^3 + 11n$ is divisible by 3
for every positive integer n.

$n^3 + 11n = (n-1)n(n+1) + 12n$.
One of three consecutive integers is a multiple of 3,
so $(n-1)n(n+1)$ is divisible by 3.
12 is a multiple of 3, so 12n is too. Hence the sum is divisible by 3.
Answer: Proved

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5 Stratified sample then probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A school has 240 students in Year 10, 60 in Year 11 and 180 in the sixth form.
A stratified sample of 24 students is taken.
(a) Find the number sampled from each group.
(b) Two students are chosen at random from the sample. Find the probability they are from the same group.

Sample sizes: $24 \times \frac{240}{480}$, $24 \times \frac{60}{480}$, $24 \times \frac{180}{480} = 12, 3, 9$

P(same) = $12 \times 11 + 3 \times 2 + 9 \times 8 / 24 \times 23$

$= \frac{210}{552} = 0.380$

Answer: (a) 12, 3, 9 (b) 0.380

6 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = e^y + 3y$.
Find dy/dx and d^2y/dx^2 when $y = 0$.

$dx/dy = e^y + 3 = 4$ when $y = 0$, so $dy/dx = \frac{1}{dx/dy} = \frac{1}{4}$

$d^2x/dy^2 = 1$ when $y = 0$

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{1}{4^3}$

Answer: $dy/dx = \frac{1}{4}$, $d^2y/dx^2 = -\frac{1}{64}$

7 Sinusoidal velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. At time t seconds its velocity is v m s^{-1}, where $v = 3\sin(\frac{\pi t}{2})$ for $t \geq 0$.

Find (a) the greatest magnitude of the acceleration of P, (b) the distance P travels between the first two times at which it is at rest. Give (b) to 3 s.f.

$a = \frac{dv}{dt} = \frac{3}{2}\pi\cos(\frac{\pi t}{2})$, so the greatest magnitude is $\frac{3}{2}\pi$

$v = 0$ when $\sin(\frac{\pi t}{2}) = 0$: $t = 0$ and $t = 2$

$s = \int$ from 0 to 2 of $3\sin(\frac{\pi t}{2})\,dt = [-\frac{3 \times 2}{\pi}\cos(\frac{\pi t}{2})]$ from 0 to 2

$= \frac{6}{\pi}(1 - (-1)) = \frac{12}{\pi}$

Answer: (a) $\frac{3}{2}\pi$ m s^{-2} (b) 3.82 m

8 Tangent to a circle and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle with equation $(x - 4)^2 + (y + 0)^2 = 169$
passes through P(−1, 12). The tangent at P meets the axes at A and B. Find the area of triangle OAB.

Centre (4, 0). Gradient of radius CP = $-\frac{12}{5}$.

Tangent gradient = $\frac{5}{12}$: $-5x + 12y = 149$.

$x = 0$: $y = \frac{149}{12}$. $y = 0$: $x = -\frac{149}{5}$.

Area = ½ × $\frac{149}{5}$ × $\frac{149}{12}$.

Answer: $\frac{22201}{120}$

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9 Partial fractions then integrate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Express $\frac{x + 6}{(x + 3)(x + 2)}$ in partial fractions.

(b) Hence find the exact value of $\int_0^1 \frac{x + 6}{(x + 3)(x + 2)}\,dx$.

Let N/D = $\frac{A}{x + 3} + \frac{B}{x + 2}$: $x + 6 \equiv A(x + 2) + B(x + 3)$

$x = -3$ gives $A = -3$; $x = -2$ gives $B = 4$

Integral = $[-3 \ln(x + 3) + 4 \ln(x + 2)]$

Substitute limits 0 and 1

Answer: (a) $-3/(x + 3) + 4/(x + 2)$; (b) $-10\ln 2 + 7\ln 3$

10 Arithmetic terms forming a geometric sequence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic sequence is 1 and its common difference d is non-zero. The 2nd, 4th and 9th terms are the first three terms of a geometric sequence. Find d and the common ratio.

Terms: $1 + d$, $1 + 3d$, $1 + 8d$.

Geometric: $(1 + 3d)^2 = (1 + d)(1 + 8d)$.

Expand: $9d^2 + 6d + 1 = 8d^2 + 9d + 1$, so $d(d - 3) = 0$.

$d \neq 0$, so $d = 3$. Ratio = $\frac{10}{4} = \frac{5}{2}$.

Answer: d = 3, ratio = $\frac{5}{2}$