Day 107 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(8\mathbf{i} - 6\mathbf{j})$ N, $(1\mathbf{i} + k\mathbf{j})$ N and $(4\mathbf{j})$ N act on a particle of mass 5 kg. The resultant is parallel to $(9\mathbf{i} - 10\mathbf{j})$.
Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant = $(9)\mathbf{i} + (-2 + k)\mathbf{j}$

Parallel to $(9\mathbf{i} - 10\mathbf{j})$: $\frac{-2 + k}{9} = \frac{-10}{9}$, so $k = -8$

R = $(9\mathbf{i} - 10\mathbf{j})$ N, so $|R| = \sqrt{(9^2 + -10^2)} = 13.45$
a = |R| ÷ 5
Answer: (a) k = −8 (b) 2.69 m s^{-2}

2 Proof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sam claims that $n^2 + n + 11$ is prime for every positive integer n.
Show that Sam is wrong by finding the smallest value of n that is a counter-example.

Test n = 1, 2, 3, ... in turn until the value is not prime.
For n = 10: $10^2 + 10 + 11 = 121$.
$121 = 11 \times 11$, which is not prime.
All smaller values of n give a prime, so n = 10 is the smallest.
Answer: n = 10 (value 121 = 11 × 11)

3 Addition formulae with exact values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is acute and B is acute. $\sin A = \frac{5}{13}$ and $\sin B = \frac{8}{17}$.
Find the exact values of cos(A + B) and tan(A − B).

$\cos A = \frac{12}{13}$, $\sin A = \frac{5}{13}$; $\sin B = \frac{8}{17}$, $\cos B = \frac{15}{17}$

$\cos(A + B) = \cos A \cos B - \sin A \sin B = \frac{140}{221}$

$\tan(A - B) = \frac{\sin(A - B)}{\cos(A - B)} = \frac{-\frac{21}{221}}{\frac{220}{221}} = -\frac{21}{220}$

Answer: $\cos(A + B) = \frac{140}{221}$, $\tan(A - B) = -\frac{21}{220}$

4 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A shell is projected from level ground with horizontal and vertical velocity components 53.9 and 63.7 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take g = 9.8 m s^{-2}.

At 45° to the horizontal, |v_y| = v_x = 53.9
Moving upwards: 63.7 − 9.8t = 53.9, so t = 1
Moving downwards: 63.7 − 9.8t = −53.9, so t = 12
Horizontal distance = 53.9 × (12 − 1)
Answer: (a) 1 s and 12 s (b) 592.9 m

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5 Regression line from summations (AQA, Edexcel, OCR, MEI)

For n = 15 pairs of values: $\Sigma x = 90$, $\Sigma y = 720$, $\Sigma x^2 = 610$, $\Sigma xy = 4180$.
(a) Find the equation of the regression line of y on x.
(b) The x values lie between 2 and 11. Estimate y when x = 11 and comment on its reliability.

$S_{xx} = 610 - \frac{90^2}{15} = 70$

$S_{xy} = 4180 - \frac{90 \times 720}{15} = -140$

$b = \frac{S_{xy}}{S_{xx}} = \frac{-140}{70} = -2$

$a = \bar{y} - b\,\bar{x} = 48 - (-2)(6) = 60$

$y = 60 - 2x$

At x = 11: y = 38. This is extrapolation (outside the data range), so may be unreliable

Answer: (a) $y = 60 - 2x$ (b) 38, unreliable (extrapolation)

6 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^4 - 9x^3 + 28x^2 - 36x + 16 < 0$.

Use the factor theorem to find roots: 1, 2, 4.

$f(x) = (x - 1)(x - 2)^2(x - 4)$.

Sign of f in each region (left to right): +, −, −, +.

Take the regions where $f < 0$ (include roots if ≤ or ≥).

Answer: $1 < x < 2$ or $2 < x < 4$

7 Equations quadratic in an exponential (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3^{2x} - 19 \times 3^x + 90 = 0$,
giving each answer exactly and, where not an integer, to 3 s.f.

Let $u = 3^x$: $u^2 - 19u + 90 = 0$

$(u - 9)(u - 10) = 0$, so $3^x = 9$ or $3^x = 10$

Take logarithms: $x = \log_3 9 = 2$; $x = \log_3 10 = 2.10$

Answer: $x = \log_3 9 = 2$; $x = \log_3 10 = 2.10$

8 Normal approximation to a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(150, 0.15).
(a) State why a normal approximation is suitable for X.
(b) Use a normal approximation to estimate P(X ≥ 36), to 3 significant figures.

np = 22.5 and nq = 127.5, both greater than 5 (n large, p not extreme)

Y ~ N(22.5, 19.125) with $\sigma = 4.373$

Apply the continuity correction:

$P(X \geq 36) \approx P(Y > 35.5) = P(Z > 2.973) = 0.00148$

Answer: 0.00148

9 Volumes of revolution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region bounded by $y = \sqrt{\sin x}$, the x-axis and the lines
$x = 0$ and $x = \pi$ is rotated through 2π about the x-axis.
Find the exact volume of the solid formed.

$V = \pi \int_0^{\pi} y^2\,dx = \pi \int_0^{\pi} (\sqrt{\sin x})^2\,dx$

Simplify the integrand, then integrate

$V = \pi \times (2)$

Answer: 2π

10 Perpendicular lines and area (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the diagram, line l_1 passes through A and B.
Line l_2 is perpendicular to l_1 at B and meets the
x-axis at C. Find the coordinates of C and the area of triangle ABC.

Gradient of $l_1 = \frac{6}{2-11} = -\frac{2}{3}$.

Gradient of $l_2 = \frac{3}{2}$. Equation: $y - 6 = \frac{3}{2}(x - 2)$.

$y = 0$: $x = -2$, so C$(-2, 0)$.

AB $= 3\sqrt{13}$, BC $= 2\sqrt{13}$: area = ½ × AB × BC.

Answer: C$(-2, 0)$; area 39