Day 103 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both
$x^2 - 5x + 4 < 0$ and $3x + 5 \leq 10$.

Quadratic: $(x - 1)(x - 4) < 0$ gives $1 < x < 4$.

Linear: $3x + 5 \leq 10$ gives $x \leq \frac{5}{3}$.

Both must hold: take the overlap of the two sets.

Answer: $1 < x \leq \frac{5}{3}$

2 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The breaking strength of a cable is N(μ, 81) with known variance 81. The mean was claimed to be 26 kN.
A random sample of 36 has mean 24.5. Test at the 10% level whether the mean is less than 26.

H_0: $\mu = 26$, H_1: $\mu < 26$. Under H_0, the sample mean ~ $N(26, \frac{81}{36})$

$z = \frac{24.5 - 26}{9 \div \sqrt{36}} = -1.000$

p-value = $P(Z < -1.000) = 0.159$

$0.159 > 10\%$, so do not reject H_0

There is insufficient evidence at the 10% level that the mean has changed as claimed

Answer: Do not reject H_0 ($z = -1.000$, p-value 0.159)

3 Logarithmic equations with ln (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2 \ln x = \ln(x + 90) + \ln 1$,
stating clearly why one root is rejected.

$\ln x^2 = \ln(1(x + 90))$, so $x^2 = 1x + 90$

$x^2 - 1x - 90 = 0$, $(x - 10)(x + 9) = 0$

$x = -9$ is rejected since ln x is undefined for $x < 0$

Answer: $x = 10$

4 Factor theorem and cubic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + ax^2 + bx - 12$, where a and b are constants.
$(x + 4)$ is a factor of f(x), and f(x) leaves remainder -18 when divided by $(x - 2)$.
Find a and b, and hence solve $f(x) = 0$.

$f(-4) = 0$: $-64 + 16a - 4b - 12 = 0$, so $16a - 4b - 76 = 0$.

$f(2) = -18$: $4a + 2b - 4 = -18$.

Solving: $a = 2$, $b = -11$.

$f(x) = x^3 + 2x^2 - 11x - 12 = (x + 4)(x + 1)(x - 3)$

Answer: $a = 2$, $b = -11$; $x = -4, -1, 3$

5 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\operatorname{cosec}(\theta - 30°) = \frac{2}{\sqrt{3}}$
for $-180° \leq \theta \leq 180°$.

Let $u = \theta - 30°$; then $\operatorname{cosec} u = \frac{2}{\sqrt{3}}$

Base solutions: $u = 120°, 60°$ (+ multiples of 360°)

$\theta = (u + 30°) \div 1$, keeping those in range

Answer: $\theta = 90°, 150°$

6 Binomial expansion (integer powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of $(x^2 - \frac{2}{x^3})^{10}$.

General term: $C(10,r)(x^2)^{10-r}(-\frac{2}{x^3})^r$.

Power of x: $2(10 - r) - 3r = 0$, so $r = 4$.

Term = $C(10,4) \times 1^6 \times (-2)^4 = 3360$.

Answer: 3360

7 Slide down a rough slope then along rough ground (CIE)

A particle starts from rest at the top of a rough slope of length 15 m ($\sin\alpha = \frac{4}{5}$, $\mu = 0.4$), then slides along rough horizontal ground with the same μ.

Use energy to find (a) its speed at the foot of the slope (3 s.f.), (b) the distance it slides on the ground. Take $g = 9.8$ m s^{-2}.

On the slope: $\frac{1}{2} m v^2 = m g \times 15 \sin\alpha - \mu m g \cos\alpha \times 15$

$v^2 = 2 \times 9.8 \times 15(\frac{4}{5} - 0.4 \times \frac{3}{5}) = 164.64$, so $v = 12.83$

On the ground: $\frac{1}{2} m v^2 = \mu m g x$, so $x = \frac{v^2}{2\mu g} = \frac{164.64}{7.84} = 21$

Answer: (a) 12.8 m s^{-1} (b) 21 m

8 Triangle in 3D: angle and area (CIE (9709); Further Maths on other boards)

The points A, B, C have coordinates (2, 5, 5), (0, 0, −2), (1, 4, −2).

Find angle ABC to 1 d.p., and the exact area of triangle ABC.

$BA = \begin{pmatrix}2\\5\\7\end{pmatrix}$, $BC = \begin{pmatrix}1\\4\\0\end{pmatrix}$; $BA \cdot BC = 22$

$|BA| = \sqrt{78}$, $|BC| = \sqrt{17}$

$\cos ABC = \frac{22}{\sqrt{78}\sqrt{17}} = 0.6042$, so $ABC = 52.8°$

$|BA \times BC| = \left|\begin{pmatrix}-28\\7\\3\end{pmatrix}\right| = \sqrt{842}$

Area $= \frac{1}{2}|BA \times BC|$

Answer: ABC = 52.8°, area = $\frac{1}{2}\sqrt{842}$

9 Coded regression line (AQA, Edexcel, OCR, MEI)

Data pairs (x, y) are coded by $u = x - 5$ and $v = \frac{y}{20}$. The regression line of v on u is $v = 10 + 1.5u$.

(a) Find the regression line of y on x in the form $y = a + bx$.

(b) Use it to estimate y when $x = 10$.

Substitute $u = x - 5$ and $v = y/20$: $\frac{y}{20} = 10 + 1.5(x - 5)$

$y = 20[10 + 1.5x - 7.5] = 20 \times 2.5 + 20 \times 1.5x$

$y = 50 + 30x$

At $x = 10$: $y = 50 + 30 \times 10 = 350$

Answer: (a) y = 50 + 30x (b) 350

10 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a point O on horizontal ground with speed 24.5 m s^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. Find (a) the greatest height, (b) the speed when it is at half of this height (3 s.f.). Take g = 9.8 m s^{-2}.

Vertical: $0 = 19.6^2 - 2 \times 9.8 \times H$, so H = 19.6

At height $\frac{H}{2}$ = 9.8: $v_y^2 = 19.6^2 - 2 \times 9.8 \times 9.8 = 192.08$

$v_x = 14.7$ (constant), so speed^2 = 216.09 + 192.08 = 408.17

Answer: (a) 19.6 m (b) 20.2 m s^{-1}