Day 10 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (7 kg) lies on a rough table with $\mu = 0.2$, and is joined by a light string over a smooth pulley (see diagram) to particle B (9 kg) on a smooth slope, $\sin\alpha = \frac{3}{5}$.

Released from rest, the string is cut when B has moved 3 m. Find the further distance A slides before stopping.

A: $T - 0.2 \times 7 \times 9.8 = 7a$. B: $9 \times 9.8 \times \frac{3}{5} - T = 9a$

Adding: $9.8(9 \times \frac{3}{5} - 0.2 \times 7) = 16a$, so $a = 2.45$

$T = 7(2.45 + 0.2 \times 9.8) = 30.87$

When cut: $v^2 = 2 \times 2.45 \times 3 = 14.7$. Then A decelerates at $\mu g = 1.96$

Further distance $= \frac{14.7}{2 \times 1.96} = 3.75$

Answer: 3.75 m

2 Sum of two independent copies of a discrete variable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X takes values 2, 3, 4, 5 with $P(X = x) = k(7 - x)$. X_1 and X_2 are independent copies of X.

(a) Find k.

(b) Find $P(X_1 + X_2 = 6)$.

(c) Find $P(X_1 = X_2)$ and $E(X_1 + X_2)$.

(a) $\Sigma P = k(5 + 4 + 3 + 2) = 14k = 1$, so $k = \frac{1}{14}$

(b) Ways to total 6: (2,4), (3,3), (4,2)

$P = \frac{5}{14} \times \frac{3}{14} + \frac{2}{7} \times \frac{2}{7} + \frac{3}{14} \times \frac{5}{14} = \frac{23}{98}$

(c) $P(X_1 = X_2) = \Sigma [P(X = x)]^2 = \frac{27}{98}$

$E(X) = \frac{22}{7}$, so $E(X_1 + X_2) = 2 \times \frac{22}{7} = \frac{44}{7}$

Answer: (a) $k = \frac{1}{14}$ (b) $\frac{23}{98}$ (c) $\frac{27}{98}$, $\frac{44}{7}$

3 Stone thrown from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 5 m s^{-1} from the top of a cliff. The top of the cliff is 146.4 m above the sea.

Find (a) the time taken for the stone to reach the sea, (b) the total distance travelled by the stone, to 3 s.f. Take $g = 9.8$ m s^{-2}.

Take upwards as positive: $-146.4 = 5t - 4.9t^2$, so $4.9t^2 - 5t - 146.4 = 0$

$t = \frac{5 + \sqrt{(5^2 + 19.6 \times 146.4)}}{9.8} = \frac{5 + 53.8}{9.8} = 6$

Height risen above the cliff top $= \frac{5^2}{(2 \times 9.8)} = 1.28$ m

Total distance $= 2 \times 1.28 + 146.4$

Answer: (a) 6 s (b) 149 m

4 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

25 recorded masses (grams) have mean 45 and standard deviation 6.

One value was recorded as 63 but should have been 71.

Find the correct mean and the correct standard deviation.

$\Sigma x = 25 \times 45 = 1125$, so corrected $\Sigma x = 1125 - 63 + 71 = 1133$

Correct mean $= \frac{1133}{25} = 45.32$

$\Sigma x^2 = 25(36 + 2025) = 51525$, corrected $\Sigma x^2 = 51525 - 3969 + 5041 = 52597$

Variance $= \frac{52597}{25} - 45.320^2 = 49.978$, $\sigma = 7.069$

Answer: mean 45.3, standard deviation 7.07

5 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{5x + 11} = x - 5$.

Square both sides: $5x + 11 = (x - 5)^2$.

$x^2 - 15x + 14 = 0$, so $x = 1$ or $x = 14$.

Check in $\sqrt{5x + 11} = x - 5$: $x = 14$ gives $9 \geq 0$ (valid).

$x = 1$ gives right-hand side $-4 < 0$, so it is rejected.

Answer: $x = 14$

6 Logarithmic equations with ln (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2 \ln x = \ln(x + 12) + \ln 1$,
stating clearly why one root is rejected.

$\ln x^2 = \ln(1(x + 12))$, so $x^2 = 1x + 12$

$x^2 - 1x - 12 = 0$, $(x - 4)(x + 3) = 0$

$x = -3$ is rejected since $\ln x$ is undefined for $x < 0$

Answer: $x = 4$

7 Quadratic sequence from recurrence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = u_n + 2n + 4$, $u_1 = 2$.

(a) Show that $u_n = an^2 + bn + c$ and find a, b, c.

(b) Find the least n for which $u_n > 1397$.

$$u_n = u_1 + \sum_{k=1}^{n-1} (2k + 4) = 2 + 2 \times \frac{(n - 1)n}{2} + 4(n - 1).$$

$= n^2 + 3n - 2$.

So $a = 1$, $b = 3$, $c = -2$.

$u_{35} = 1328 \leq 1397$ and $u_{36} = 1402 > 1397$.

Answer: (a) $a = 1$, $b = 3$, $c = -2$ (b) $n = 36$

8 Points of inflection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coordinates of the points of inflection of

$y = \frac{6}{x^2 + 3}$.

$$dy/dx = -\frac{12x}{(x^2 + 3)^2}$$

$$d^2y/dx^2 = \frac{12(3x^2 - 3)}{(x^2 + 3)^3}$$

Zero when $x^2 = 1$, so $x = \pm 1$, and the sign changes either side

$y = \frac{6}{4} = \frac{3}{2}$

Answer: $(1, \frac{3}{2})$ and $(-1, \frac{3}{2})$

9 Parametric curve and line intersection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has parametric equations
$x = t$, $y = 3t^2$. The line $y = 15x - 18$
meets C at A and B. Find the exact length AB.

Substitute: $3t^2 = 15t - 18$.

$3t^2 - 15t + 18 = 0 \Rightarrow t = 3$ or $t = 2$.

A(3, 27), B(2, 12).

$AB^2 = 1^2 + 15^2 = 226$.

Answer: AB = $\sqrt{226}$

10 Identity then equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Show that $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} \equiv \tan^2\theta$.

(b) Hence solve $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \frac{1}{3}$ for $0 \leq \theta < 360°$ (1 d.p.).

$\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \frac{2\sin^2\theta}{2\cos^2\theta} = \tan^2\theta$

$\tan^2\theta = \frac{1}{3}$, so $\tan\theta = \pm\sqrt{\frac{1}{3}}$

$\theta = 30.0°$, then $180° - \theta$, $180° + \theta$, $360° - \theta$

Answer: θ = 30.0°, 150.0°, 210.0°, 330.0°