Day 99 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Driving force and resistance (All boards)

A car of mass 500 kg is driven along a straight horizontal road by a driving force of 875 N against a constant resistance of 500 N.

Find the acceleration of the car.

Resultant force = 875 − 500 = 375 N

F = ma: 375 = 500 a

Answer: $0.75\ \text{m s}^{-2}$

2 Venn diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities for events A and B.

Find P(A | B).

$P(A \mid B) = P(A \cap B) \div P(B)$

$= 0.2 \div (0.2 + 0.25) = 0.2 \div 0.45$

Answer: $\frac{4}{9} = 0.444$

3 Solve a^x = b using logarithms (All boards)

Solve $5^{x+1} = 100$.

Give your answer to 3 significant figures.

Take logs: $(x + 1) \log 5 = \log 100$

$x + 1 = 2.861$, so $x = 1.86$

Answer: $x = 1.86$

4 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 - 1$, $x \geq 0$.

Find $f^{-1}(x)$ and state its domain.

$y = x^2 - 1$, so $x^2 = y + 1$

$x \geq 0$, so $x = +\sqrt{y + 1}$

The domain of f^{-1} is the range of f: $y \geq -1$

Answer: $f^{-1}(x) = \sqrt{x + 1}$, $x \geq -1$

5 Quotient rule (All boards)

Use the quotient rule to find the derivative of

$y = \frac{x}{x + 2}$.

Quotient rule: if $y = \frac{u}{v}$ then $dy/dx = \frac{u'v - uv'}{v^2}$

Differentiate the numerator and denominator, then substitute and simplify.

Answer: $dy/dx = \frac{2}{(x + 2)^2}$

6 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x + 3)^2 + (y - 3)^2 = 16$.

Determine whether the point P(−5, −5) is inside, outside or on the circle.

Distance^2 from the centre (−3, 3) to P:

$(-5 - (-3))^2 + (-5 - 3)^2 = 4 + 64 = 68$

$r^2 = 16$, and $68 > 16$.

Answer: P is outside the circle

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7 Maximum value from R form (All boards)

f(x) = 15 sin x + 8 cos x + 1.
Write f(x) in the form R sin(x + α) + 1, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{15^2 + 8^2} = 17$, $\tan \alpha = \frac{8}{15}$ so $\alpha = 28.1°$

f(x) = 17 sin(x + 28.1°) + 1
sin(x + α) has maximum value 1, so f(max) = 17 + 1
Answer: f(x) = 17 sin(x + 28.1°) + 1; maximum 18

8 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 20 kg and length 8 m rests horizontally on supports at A and B. A load of mass 20 kg is placed on the beam 2 m from A. Take g = 9.8 m s^{-2}. Find the reaction at B.

Moments about A:
$R_B \times 8 = 20 \times 9.8 \times 4 + 20 \times 9.8 \times 2$
$R_B \times 8 = 1176$
Answer: 147 N

9 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 7 terms of the geometric series 128 + 64 + 32 + ...

$a = 128$, $r = \frac{1}{2}$, $n = 7$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{128(1 - (\frac{1}{2})^7)}{1 - \frac{1}{2}}$$

= 254
Answer: 254

10 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(135, 8^2)$. Given that P(X < a) = 0.8, find a to 3 s.f.

P(Z < z) = 0.8, so z = 0.8416 (from the inverse normal function)
a = 135 + 0.8416 × 8
Answer: 142