Day 98 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 9 terms of the geometric series 6 + 24 + 96 + ...

$a = 6$, $r = 4$, $n = 9$

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{6(4^9 - 1)}{4 - 1}$

$= 524286$

Answer: 524286

2 Coding (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The values x are coded using $y = (x - 40) \div 4$.

The coded values have mean $\bar{y} = 2.2$ and standard deviation 2.4.

Find the mean and standard deviation of x.

$x = 40 + 4y$

Mean: $40 + 4 \times 2.2 = 48.8$

Standard deviation: $4 \times 2.4 = 9.6$ (adding 40 has no effect)

Answer: mean = 48.8, standard deviation = 9.6

3 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points A and B, 60 m apart, and cycle directly towards each other at constant speeds of 3 m s^{-1} and 7 m s^{-1}. Find the time until they meet.

Closing speed = $3 + 7 = 10$ m s^{-1}

Time = $\frac{60}{10}$

Answer: 6 s

4 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = -8\mathbf{i} - 6\mathbf{j}$.

Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{8^2 + 6^2} = 10$

(b) Acute angle = $\tan^{-1}(\frac{6}{8}) = 36.9°$

p is in the third quadrant, so the angle is 216.9°

Answer: (a) 10 (b) 216.9°

5 Parametric equations to Cartesian form (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A line has parametric equations

$x = 3t - 5$, $y = 2t + 2$.

Find its equation in the form $y = mx + c$.

From $x = 3t - 5$: $t = \frac{x + 5}{3}$

$y = 2 \times \frac{x + 5}{3} + 2$

Answer: $y = \frac{2}{3}x + \frac{16}{3}$

6 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train starts from rest and accelerates uniformly at 2 m s^{-2} for 6 s, then travels at constant speed for a further 30 s. Find the total distance travelled.

Speed reached: $v = 2 \times 6 = 12$ m s^{-1}

Stage 1: $s = \frac{1}{2} \times 12 \times 6 = 36$

Stage 2: $s = 12 \times 30 = 360$

Answer: 396 m

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7 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 3x + k$ is a tangent to the curve
$y = x^2 - 3x - 4$.
Find the value of k.

$x^2 - 3x - 4 = 3x + k$
$x^2 - 6x + (-4 - k) = 0$
A tangent touches once: discriminant = 0
$(-6)^2 - 4(-4 - k) = 0$
Answer: $k = -13$

8 Identity then solve ($\sin^2$ to cos) (All boards)

Solve $-2\cos^2 x + \sin x + 2 = 0$
for $0° \leq x \leq 360°$.

Use $\cos^2 x = 1 - \sin^2 x$: $2\sin^2 x + \sin x = 0$

Factorise and solve: $\sin x = 0$ or $\sin x = -\frac{1}{2}$

$x = 0°, 180°, 210°, 330°, 360°$
Answer: $x = 0°, 180°, 210°, 330°, 360°$

9 Normal approximation to the binomial (CIE)

$X \sim B(80, 0.5)$. Use a normal approximation to find $P(X \leq 36)$, to 3 s.f.

$np = 40$ and $npq = 20$, so use $Y \sim N(40, 20)$
Continuity correction: $P(Y < 36.5)$
$z = (36.5 - 40) \div \sqrt{20}$
Answer: 0.217

10 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = \sqrt{x + 3}$ from $x = 0$ to $x = 6$ divided into 4 strips of equal width.
Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

h = 1.5; y values: 1.732, 2.121, 2.449, 2.739, 3.000

Area $\approx \frac{1.5}{2}[1.732 + 3.000 + 2(7.309)]$

$= 14.5$
Answer: 14.5