Day 97 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $5\,e^x + 5 \ln x$

(b) $e^{5x} + \sin 5x$

(a) $dy/dx = 5\,e^x + \frac{5}{x}$

(b) $dy/dx = 5\,e^{5x} + 5 \cos 5x$

Answer: (a) $5\,e^x + \frac{5}{x}$ (b) $5\,e^{5x} + 5 \cos 5x$

2 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $-2x^3 + x^2 - 3x + 6$ is divided by $(2x - 3)$.

Put $2x - 3 = 0$: $x = \frac{3}{2}$

Remainder $= f(\frac{3}{2}) = (-\frac{27}{4}) + \frac{9}{4} + (-\frac{9}{2}) + 6$

$= -3$

Answer: -3

3 Poisson distribution (CIE)

Cars pass a checkpoint at an average rate of 0.5 per hour, at random.
Find the probability of exactly 1 in 3 hours, to 3 s.f.

Mean for 3 hours $= 3 \times 0.5 = 1.5$

$P(X = 1) = e^{-1.5} \times 1.5^1 \div 1!$

Answer: 0.335

4 Vertical motion under gravity: thrown upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is thrown vertically upwards with speed 49 m s^{-1}. Take g = 9.8 m s^{-2}.
(a) Find the time taken to reach its greatest height.
(b) Find the greatest height reached.

(a) $v = u + at$: $0 = 49 - 9.8t$, so $t = 5$

(b) $v^2 = u^2 + 2as$: $0 = 49^2 - 2 \times 9.8 \times s$

$s = \frac{2401}{19.6}$

Answer: (a) 5 s (b) 122.5 m

5 Differentiate tan, sec, cosec, cot (AQA, Edexcel, OCR, MEI, Eduqas)

Differentiate $y = \sec 2x$ with respect to x.

Use the standard result: $d/dx(\sec kx) = k \sec kx \tan kx$

Answer: $dy/dx = 2 \sec 2x \tan 2x$

6 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $6x + 9y = 97$.

Assume that integers x and y exist with $6x + 9y = 97$.
Then $3(2x + 3y) = 97$, so 97 would be a multiple of 3.
But $97 = 3 \times 32 + 1$, so it is not a multiple of 3.
This is a contradiction, so no such integers exist.

Answer: Proved

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7 Rough slope: limiting equilibrium or sliding (All boards)

A crate is on the point of sliding down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$. Find the coefficient of friction.

Perpendicular: $R = mg\cos\alpha$
Parallel (limiting): $F = mg\sin\alpha$
$\mu = \frac{F}{R} = \tan\alpha$

Answer: $\mathbf{\frac{3}{4}}$

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x + k$ is a tangent to the curve
$y = x^2 - 3x - 3$.
Find the value of k.

$x^2 - 3x - 3 = -x + k$
$x^2 - 2x + (-3 - k) = 0$
A tangent touches once: discriminant = 0
$(-2)^2 - 4(-3 - k) = 0$
Answer: $\mathbf{k = -4}$

9 Double angle: find sin 2A and cos 2A (All boards)

A is acute and $\cos A = \frac{15}{17}$.

Find the exact values of sin 2A and cos 2A.

$\sin A = \sqrt{1 - \cos^2 A} = \frac{8}{17}$

$\sin 2A = 2\sin A\cos A = 2 \times \frac{8}{17} \times \frac{15}{17} = \frac{240}{289}$

$\cos 2A = 2\cos^2 A - 1 = 2 \times \frac{225}{289} - 1 = \frac{161}{289}$

Answer: $\mathbf{\sin 2A = \frac{240}{289}, \cos 2A = \frac{161}{289}}$

10 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.4. If so, the probability that the patient recovers is 0.8; otherwise it is 0.3. Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = 0.4 × 0.8 = 0.32; P(not first and second) = 0.6 × 0.3 = 0.18
P(second) = 0.32 + 0.18 = 0.5
P = 0.32 ÷ 0.5

Answer: $\mathbf{\frac{16}{25} = 0.64}$