Day 96 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Area between a line and a curve (All boards)

The curve $y = 7x - x^2$ and the line $y = 2x$ meet at the origin and at P.
Find the area enclosed between them.

$7x - x^2 = 2x$ gives $5x - x^2 = 0$, so $x = 0$ or $x = 5$

Area $= \int_0^5 (5x - x^2)\,dx = \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]$ from 0 to 5

$= \frac{5^3}{2} - \frac{5^3}{3} = \frac{125}{6}$

Answer: $\mathbf{\frac{125}{6}}$

2 Gradient and parallel or perpendicular lines (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -4x - 5$.
Find the equation of the line perpendicular to l that passes through $(5, -1)$.

Gradient of l is -4, so the perpendicular gradient is $\frac{1}{4}$.

$y - (-1) = \frac{1}{4}(x - 5)$

$y = \frac{1}{4}x - \frac{9}{4}$

Answer: $\mathbf{y = \frac{1}{4}x - \frac{9}{4}}$

3 Sampling methods (AQA, Edexcel, OCR, MEI, Eduqas)

A reporter asks the first 20 people who walk out of a hospital for their views.
Name the sampling method used.

This is opportunity sampling: people are chosen only because they are available at the time.
Answer: opportunity sampling

4 Definite integrals (exact) (All boards)

Evaluate

$\int_0^2 (x^2 + 4)\,dx$.

$\left[\frac{x^3}{3} + 4x\right]$ from 0 to 2

$= \frac{8}{3} + 8 = \frac{32}{3}$

Answer: $\mathbf{\frac{32}{3}}$

5 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which
$x^2 + kx + 36 = 0$ has two distinct real roots.

$b^2 - 4ac > 0$

$k^2 - 144 > 0$, so $k^2 > 144$

Answer: $k < -12$ or $k > 12$

6 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = k(x + 1)$ for $0 \le x \le 1$, and 0 otherwise.
Find the value of k.

$\int$ from 0 to 1 of $k(x + 1)\,dx = 1$

$k\left[\frac{x^2}{2} + 1x\right]$ from 0 to 1 $= k(\frac{3}{2}) = 1$

Answer: $\mathbf{k = \frac{2}{3}}$

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7 Separable differential equations (All boards)

Given that $dy/dx = \frac{x}{y}$ and $y = 2$ when $x = 0$,

find y^2 in terms of x.

$y\,dy = x\,dx$, so $\frac{y^2}{2} = \frac{x^2}{2} + c$

$x = 0, y = 2$: $c = \frac{4}{2}$

$y^2 = x^2 + 4$

Answer: $y^2 = x^2 + 4$

8 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A ball is projected from level ground with velocity $(14\mathbf{i} + 19.6\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take $g = 9.8$ m s^{-2}.
(a) Find the time of flight.
(b) Find the horizontal range.

(a) Vertically: $0 = 19.6\,t - 4.9t^2$

$t = \frac{19.6}{4.9} = 4$

(b) Range $= 14 \times 4$

Answer: (a) 4 s (b) 56 m

9 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:
$y = x^2 + 6x - 9$, $y = 3x - 5$.

$x^2 + 6x - 9 = 3x - 5$

$x^2 + 3x - 4 = 0$

$(x - 1)(x + 4) = 0$

Use $y = 3x - 5$ to find each y.

Answer: $x = -4$, $y = -17$ and $x = 1$, $y = -2$

10 Velocity-time graph: acceleration and distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a car over 46 seconds as it moves in a straight line.
(a) Find its acceleration in the first 10 s.
(b) Find the total distance travelled.

(a) acceleration = gradient $= \frac{10}{10}$

(b) distance = area of trapezium

$= \frac{1}{2}(30 + 46) \times 10$

Answer: (a) 1 m s^{-2} (b) 380 m