Day 93 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplify using identities (All boards)

Simplify $1 + \tan^2 x$.

Use $1 + \tan^2 x = \sec^2 x$

Answer: $\sec^2 x$

2 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 + 1$, $x \geq 0$.

Find $f^{-1}(x)$ and state its domain.

$y = x^2 + 1$, so $x^2 = y - 1$

$x \geq 0$, so $x = +\sqrt{y - 1}$

The domain of f^{-1} is the range of f: $y \geq 1$

Answer: $f^{-1}(x) = \sqrt{x - 1}$, $x \geq 1$

3 Resolving a force into components (All boards)

A string pulls a box with a force of 10 N at an angle of 45° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal = $10 \cos 45°$

Vertical = $10 \sin 45°$

Answer: horizontal 7.07 N, vertical 7.07 N

4 Addition formulae: exact values (All boards)

By writing 105° as 60° + 45°, find the exact value of cos 105°.

$\cos(60° + 45°) = \cos 60° \cos 45° - \sin 60° \sin 45°$

$= \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$

Simplify: $-\frac{\sqrt{6} - \sqrt{2}}{4}$

Answer: $-\frac{\sqrt{6} - \sqrt{2}}{4}$

5 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Use proof by exhaustion to show that $n^3 + 5n$ is divisible by 6 for every integer n with $1 \leq n \leq 4$.

Substitute each value of n:

n = 1: $6 = 6 \times 1$; n = 2: $18 = 6 \times 3$

n = 3: $42 = 6 \times 7$; n = 4: $84 = 6 \times 14$

Every value is a multiple of 6, so the statement is true for $1 \leq n \leq 4$.

Answer: Shown

6 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 8^2)$. H_0: $\mu = 175$, H_1: $\mu < 175$.

A sample of 9 has mean $\bar{x} = 171.27$.

Test at the 5% level. Critical value: $z = -1.645$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (171.27 - 175) \div (8 \div 3)$

$z = -1.40$

-1.40 is outside the critical region, so do not reject H_0

Answer: $z = -1.40$; do not reject H_0

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7 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both
$2x - 5 < -13$ and $x^2 + 9x + 18 < 0$.

$2x - 5 < -13$ gives $x < -4$.
$x^2 + 9x + 18 < 0$: $(x + 6)(x + 3) < 0$, so $-6 < x < -3$.
Both are true where the two ranges overlap.
Answer: $-6 < x < -4$

8 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(30, p)$. H_0: $p = 0.6$, H_1: $p > 0.6$.
Find the critical region for a test at the 10% significance level.

$P(X \geq 22) = 0.0940 \leq 0.1$
$P(X \geq 21) = 0.176 > 0.1$
Answer: $X \geq 22$

9 Integration by parts (All boards)

Use integration by parts to find
$\int x \ln x \, dx$.

$u = \ln x$, $dv/dx = x$; $du/dx = \frac{1}{x}$, $v = \frac{x^2}{2}$
$\int u \, (dv/dx) \, dx = uv - \int v \, (du/dx) \, dx$
Evaluate the remaining integral and simplify
Answer: $\frac{x^2}{2} \ln x - \frac{x^2}{4} + c$

10 Rough slope: limiting equilibrium or sliding (All boards)

A particle is on the point of sliding down a rough plane inclined at α to the horizontal, where $\tan \alpha = \frac{8}{15}$. Find the coefficient of friction.

Perpendicular: $R = mg \cos \alpha$
Parallel (limiting): $F = mg \sin \alpha$
$\mu = \frac{F}{R} = \tan \alpha$
Answer: $\frac{8}{15}$