Day 92 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Amplitude and period from a graph (diagram) (All boards)

The diagram shows the graph of $y = a \sin bx$ for $0° \leq x \leq 360°$, where a and b are positive integers.
Find a and b.

The maximum value of y is 3, so a = 3

The graph completes 4 full cycles in 360°, so the period is 90° and $b = \frac{360}{90} = 4$

Answer: a = 3, b = 4

2 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form $px + qy = r$ with integers p, q and r, (b) the length AB.

Gradient $= \frac{6 - 0}{0 - (-3)} = 2$

Using $y = mx + c$ with c = 6: $y = 2x + 6$, so $2x - y = -6$

$AB^2 = 3^2 + 6^2 = 45$

Answer: (a) $2x - y = -6$ (b) $3\sqrt{5}$

3 Energy: falling object (potential and kinetic energy) (CIE)

A particle of mass 4 kg is released from rest and falls 19.6 m. Take $g = 9.8$ m s^{-2}. Use energy to find (a) the loss in potential energy (3 s.f.), (b) the speed of the particle after falling 19.6 m.

(a) Loss in PE $= mgh = 4 \times 9.8 \times 19.6 = 768.32$

(b) $\frac{1}{2} \times 4 \times v^2 = 768.32$

$v^2 = 384.16$

Answer: (a) 768 J (b) 19.6 m s^{-1}

4 Frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a histogram, the class 0 to 5 has frequency 25 and is drawn with a bar of height 2.5 cm.
The class 5 to 15 has frequency 40. Find the height of its bar.

Frequency density = frequency ÷ class width
First bar: 25 ÷ 5 = 5, so 2.5 cm represents 5
Second bar: 40 ÷ 10 = 4

Height $= 2.5 \times \frac{4}{5}$

Answer: 2 cm

5 Solve a logarithmic equation (two logs) (All boards)

Solve $\log x + \log (x - 3) = \log 18$, $x > 3$.

$\log [x(x - 3)] = \log 18$, so $x^2 - 3x = 18$
$x^2 - 3x - 18 = 0$, $(x - 6)(x + 3) = 0$
$x = 6$ or $x = -3$; $x > 3$, so $x = 6$
Answer: x = 6

6 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:
$y = x^2 - 9$, $y = -2x - 6$.

$x^2 - 9 = -2x - 6$
$x^2 + 2x - 3 = 0$
$(x + 3)(x - 1) = 0$
Use $y = -2x - 6$ to find each y.
Answer: x = −3, y = 0 and x = 1, y = −8

7 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(25, p)$. H_0: $p = 0.1$, H_1: $p \neq 0.1$. In a sample, $x = 7$.

Test at the 5% significance level.

Two-tailed test, so compare with $0.05 \div 2 = 0.025$

$P(X \geq 7) = 0.00948$

$0.00948 < 0.025$, so reject H_0

Answer: Reject H_0 ($P(X \geq 7) = 0.00948$)

8 Maximum friction on a horizontal surface (All boards)

A box of mass 20 kg rests on rough horizontal ground. The coefficient of friction is 0.5.

Taking $g = 9.8$ m s^{-2}, find the maximum friction force that can act on the box.

$R = mg = 20 \times 9.8 = 196$

$F_{max} = \mu R = 0.5 \times 196$

Answer: 98 N

9 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ax^n$. A graph of $\log_{10} y$ against $\log_{10} x$ is a straight line with gradient $\frac{3}{2}$ and intercept 0.8.

Find n and a, giving a to 3 significant figures.

$\log y = \log a + n \log x$

Gradient $n = \frac{3}{2}$

$\log a = 0.8$ so $a = 10^{0.8} = 6.31$

Answer: $n = \frac{3}{2}$, $a = 6.31$

10 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^3 - 4x - 1$.

C is reflected in the x-axis. Find the equation of the image, in the form $y = ...$

Reflection in the x-axis: y becomes $-y$.

$-y = x^3 - 4x - 1$

$y = -x^3 + 4x + 1$

Answer: $y = -x^3 + 4x + 1$