Day 91 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Vertical motion under gravity: thrown upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A rock is thrown vertically upwards with speed 29.4 m s^{-1}. Take g = 9.8 m s^{-2}.
(a) Find the time taken to reach its greatest height.
(b) Find the greatest height reached.

(a) v = u + at: 0 = 29.4 − 9.8t, so t = 3
(b) $v^2 = u^2 + 2as$: $0 = 29.4^2 - 2 \times 9.8 \times s$
$s = \frac{864.36}{19.6}$
Answer: (a) 3 s (b) 44.1 m

2 Solve basic trig equation (exact, degrees) (All boards)

Solve $\tan x = \frac{\sqrt{3}}{3}$ for $0° \leq x \leq 360°$.

Reference angle: $\tan 30° = \frac{\sqrt{3}}{3}$
tan is positive in the first and third quadrants.
x = 30°, 210°
Answer: x = 30°, 210°

3 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $x^2 + 10x + 2$ in the form $(x + p)^2 + q$, and state the coordinates of the minimum point of $y = x^2 + 10x + 2$.

$x^2 + 10x = (x + 5)^2 - 25$
$x^2 + 10x + 2 = (x + 5)^2 - 23$
The minimum is where $(x + 5)^2 = 0$, i.e. x = −5
Answer: $(x + 5)^2 - 23$; minimum (−5, −23)

4 Chain rule: gradient at a point (All boards)

Find the gradient of the curve $y = e^{5x - 15}$
at the point where x = 3.

Use the chain rule: $dy/dx = 5\,e^{5x - 15}$
At x = 3: dy/dx = 5
Answer: 5

5 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 3x^2 - 4x + 12$
(a) Show that (x − 2) is a factor of f(x).
(b) Factorise f(x) completely.

f(2) = 8 + (−12) + (−8) + 12 = 0, so (x − 2) is a factor.
$f(x) = (x - 2)(x^2 - x - 6)$
$x^2 - x - 6 = (x + 2)(x - 3)$
Answer: (b) f(x) = (x + 2)(x − 2)(x − 3)

6 Particle on a smooth table connected to a hanging particle (All boards)

A 3 kg particle on a smooth horizontal table is joined by a light string over a smooth pulley to a hanging 2 kg particle, as shown. The system is released from rest. Take g = 9.8 m s^{-2}. Find the tension.

Table particle: T = 3 a
Hanging particle: 2 × 9.8 − T = 2 a
Adding: 19.6 = 5 a
Answer: 11.76 N

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7 Normal approximation to the binomial (CIE)

X ~ B(80, 0.5). Use a normal approximation to find P(X ≥ 42), to 3 s.f.

np = 40 and npq = 20, so use Y ~ N(40, 20)
Continuity correction: P(Y > 41.5)
$z = (41.5 - 40) \div \sqrt{20}$
Answer: 0.369

8 Quotient rule (All boards)

Use the quotient rule to find the derivative of

$y = \frac{e^x}{4x + 1}$.

Quotient rule: if $y = \frac{u}{v}$ then $dy/dx = \frac{u'v - uv'}{v^2}$

Differentiate the numerator and denominator, then substitute and simplify.

Answer: $\mathbf{dy/dx = \frac{e^x(4x + -3)}{(4x + 1)^2}}$

9 Transforming a graph with a labelled turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). The maximum point is shown. Write down the coordinates of the maximum point on the curve with equation
(a) y = 2f(x) (b) y = f(x + 3)

(a) y = 2f(x): the maximum point moves to (−4, 6).
(b) y = f(x + 3): the maximum point moves to (−7, 3).
Answer: (a) (−4, 6) (b) (−7, 3)

10 Large data set: handling readings (AQA, Edexcel, OCR, MEI, Eduqas)

Seven daily sunshine readings (hours) from the Large Data Set are: 5, n/a, 4, 0.8, n/a, 1, 0.8.
Here 'n/a' means no reading was recorded.
Leaving these out, find the mean to 3 s.f.

Leave out the 2 'n/a' readings, leaving 5 values
Total = 11.6
Mean = 11.6 ÷ 5
Answer: 2.32 hours