Day 90 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Regression: interpreting coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

A model relates the temperature, T °C, of a cooling drink to the time, t minutes, since pouring by $T = 73 - 2.3t$. Interpret the value 73 in this context.

The constant is the predicted value of T when t = 0.
So the predicted T at t = 0 is 73 °C.
Answer: predicted T is 73 °C when t = 0

2 Parallel vectors (All boards)

$\mathbf{a} = 4\mathbf{i} + 6\mathbf{j}$ and $\mathbf{b} = 8\mathbf{i} + k\mathbf{j}$, where k is a constant.
Given that a and b are parallel, find the value of k.

Parallel: $\mathbf{b} = \lambda\mathbf{a}$, so $8 = \lambda \times 4$, giving $\lambda = 2$
$k = \lambda \times 6 = 12$
Answer: k = 12

3 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points P and Q, 96 m apart, and cycle directly towards each other at constant speeds of 7 m s^{-1} and 9 m s^{-1}. Find the time until they meet.

Closing speed = 7 + 9 = 16 m s^{-1}
Time = $\frac{96}{16}$
Answer: 6 s

4 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the first four terms, in ascending powers of x, of the expansion of $(1 + 4x)^7$.

$(1 + y)^7 = 1 + 7y + 21y^2 + 35y^3 + \ldots$ with $y = 4x$
$= 1 + 7(4x) + 21(4x)^2 + 35(4x)^3$
Answer: $1 + 28x + 336x^2 + 2240x^3$

5 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity
$(3\mathbf{i} + 4\mathbf{j})$ m s^{-1}. It is at the point with position vector $(2\mathbf{i} - \mathbf{j})$ m at t = 0. Find the time when its position vector is $(20\mathbf{i} + 23\mathbf{j})$ m.

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
$(20\mathbf{i} + 23\mathbf{j}) = (2\mathbf{i} - \mathbf{j}) + t(3\mathbf{i} + 4\mathbf{j})$
Equating i components: $20 = 2 + 3t$
Answer: 6 s

6 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = 2x^3 - 10x^2 + 4x + 8$ by $(x - 4)$.
Find the quotient and the remainder.

Use synthetic division with a = 4:
Bottom row: 2, −2, −4, −8
Quotient: $2x^2 - 2x - 4$
Remainder: −8
Answer: Quotient $2x^2 - 2x - 4$, remainder −8

7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is N(μ, 8^2). H_0: $\mu = 100$, H_1: $\mu > 100$.

A sample of 49 has mean $\bar{x} = 102.17$.
Test at the 5% level. Critical value: $z = 1.645$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (102.17 - 100) \div (8 \div 7)$
$z = 1.90$
1.90 is inside the critical region, so reject H_0
Answer: $z = 1.90$; reject H_0

8 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ax^n$. A graph of $\log_{10} y$ against $\log_{10} x$ is a straight line with gradient 2 and intercept 1.3.
Find n and a, giving a to 3 significant figures.

$\log y = \log a + n \log x$
Gradient $n = 2$
$\log a = 1.3$ so $a = 10^{1.3} = 20.0$
Answer: $n = 2$, $a = 20.0$

9 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.
Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is $(3, -4)$, so $y = a(x - 3)^2 - 4$.
The curve passes through (0, 5):
$5 = a(-3)^2 - 4$, so $9 = 9a$
$a = 1$
Answer: $y = (x - 3)^2 - 4$

10 Integrate using double angle formulae (All boards)

Use a double angle formula to find
$\int \sin 2x \cos 2x \, dx$.

$\sin 2\theta = 2 \sin \theta \cos \theta$, so $\sin 2x \cos 2x = \frac{1}{2} \sin 4x$
Integrate term by term
Answer: $-\frac{\cos 4x}{8} + c$