Day 9 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Arc length and perimeter of a sector (All boards)

A sector of a circle has radius 4 cm and angle 1.6 radians.
Find (a) the arc length, (b) the perimeter of the sector.

(a) Arc length = $r\theta = 4 \times 1.6 = 6.40$ cm
(b) Perimeter = arc + 2r = 6.40 + 8 = 14.4 cm
Answer: (a) 6.40 cm (b) 14.4 cm

2 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $-2x^2 - 8x - 13$ in the form $a(x + p)^2 + q$.

Take out a factor -2: $-2(x^2 + 4x + \frac{13}{2})$

$x + 4 = (x + 2)^2 - 4$
$-2[(x + 2)^2 - 4] - 13 = -2(x + 2)^2 - 5$
Answer: $-2(x + 2)^2 - 5$

3 Equilibrium of forces given as vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(p\mathbf{i} + q\mathbf{j})$ N, $(4\mathbf{i} - \mathbf{j})$ N and $(2\mathbf{i})$ N act on a particle which is in equilibrium. $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors. Find the values of p and q.

Sum of forces = 0:
i components: p + (4) + (2) = 0
j components: q + (−1) + (0) = 0
Answer: p = −6, q = 1

4 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic series has first term 9, last term 117 and 19 terms. Find the sum of the series.

$S_n = \frac{n}{2}(a + l) = \frac{19}{2}(9 + 117)$
$= 1197$
Answer: 1197

5 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = k(x + 3)$ for $0 \le x \le 1$, and 0 otherwise.
Find the value of k.

$\int$ from 0 to 1 of $k(x + 3)\,dx = 1$
$k[\frac{x^2}{2} + 3x]$ from 0 to 1 $= k(\frac{7}{2}) = 1$
Answer: $k = \frac{2}{7}$

6 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A ball is projected from level ground with velocity $(14\mathbf{i} + 9.8\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the horizontal range.

(a) Vertically: $0 = 9.8\,t - 4.9t^2$
$t = \frac{9.8}{4.9} = 2$
(b) Range = 14 × 2
Answer: (a) 2 s (b) 28 m

7 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{-2x + 4}{(x - 5)(x - 4)}$ in partial fractions.

Write $\frac{-2x + 4}{(x - 5)(x - 4)} = \frac{A}{(x - 5)} + \frac{B}{(x - 4)}$

$-2x + 4 = A(x - 4) + B(x - 5)$

$x = 5$: $-6 = A(1)$ so $A = -6$

$x = 4$: $-4 = B(-1)$ so $B = 4$

Answer: $\mathbf{\frac{-6}{(x - 5)} + \frac{4}{(x - 4)}}$

8 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of
(a) $\arccos(-1)$ (b) $\arctan(1)$

(a) $y = \arccos(-1)$ means $\cos y = -1$ with $0 \leq y \leq \pi$: $y = \pi$

(b) $y = \arctan(1)$ means $\tan y = 1$ with $-\frac{\pi}{2} < y < \frac{\pi}{2}$: $y = \frac{\pi}{4}$

Answer: (a) $\boldsymbol{\pi}$ **(b)** $\mathbf{\frac{\pi}{4}}$

9 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: $p = 0.1$, H_1: $p \neq 0.1$. In a sample, $x = 4$.
Test at the 10% significance level.

Two-tailed test, so compare with $0.1 \div 2 = 0.05$

$P(X \geq 4) = 0.133$

$0.133 > 0.05$, so do not reject H_0

Answer: Do not reject $\mathbf{H_0}$ **(**$\mathbf{P(X \geq 4) = 0.133}$**)**

10 Separable differential equations (All boards)

Given that $dy/dx = 4xy$ and $y = 3$ when $x = 0$,
find y in terms of x.

$\frac{1}{y}\, dy = 4x\, dx$, so $\ln y = 2x^2 + c$

$x = 0$, $y = 3$: $c = \ln 3$

$y = 3\, e^{2x^2}$

Answer: $\mathbf{y = 3\, e^{2x^2}}$