Day 88 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form px + qy = r with integers p, q and r, (b) the length AB.

Gradient = $\frac{-4-0}{0-6} = \frac{2}{3}$

Using y = mx + c with c = −4: y = $\frac{2}{3}$x − 4, so 2x − 3y = 12

$AB^2 = 6^2 + 4^2 = 52$

Answer: (a) 2x − 3y = 12 (b) $2\sqrt{13}$

2 Standard deviation from summary data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 20 values, $\Sigma x = 500$ and $\Sigma x^2 = 12820$.
Find the mean and the standard deviation.

Mean = 500 ÷ 20 = 25

Variance = $\frac{12820}{20} - 25^2 = 641 - 625 = 16$

Standard deviation = $\sqrt{16}$

Answer: mean = 25, standard deviation = 4

3 Equation of a circle from a diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a circle with the centre marked.
The circle touches the x-axis. Find the equation of the circle.

The circle touches the x-axis, so the radius equals the y-coordinate of the centre: r = 3.

Centre (4, 3): $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x - 4)^2 + (y - 3)^2 = 9$

4 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ax^n$. A graph of $\log_{10} y$ against $\log_{10} x$ is a straight line with gradient 4 and intercept 1.1.
Find n and a, giving a to 3 significant figures.

log y = log a + n log x
Gradient n = 4
log a = 1.1 so $a = 10^{1.1} = 12.6$

Answer: n = 4, a = 12.6

5 Rough slope: limiting equilibrium or sliding (All boards)

A crate is on the point of sliding down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{8}{15}$. Find the coefficient of friction.

Perpendicular: R = mg cos α
Parallel (limiting): F = mg sin α
$\mu = \frac{F}{R} = \tan\alpha$

Answer: $\frac{8}{15}$

6 Area between a line and a curve (All boards)

The curve $y = x^2$ and the line y = 4x meet at the origin and at P.
Find the area enclosed between them.

$x^2 = 4x$ gives x = 0 or x = 4

Area = $\int_0^4 (4x - x^2)\,dx = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]$ from 0 to 4

$= \frac{4^3}{2} - \frac{4^3}{3} = \frac{32}{3}$

Answer: $\frac{32}{3}$

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7 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

When $x^3 + kx^2 + 2x + 59$ is divided by $(x + 3)$, the remainder is -1. Find the value of k.

$f(-3) = -1$
$-27 + 9k + 53 = -1$
$9k = -27$
Answer: $k = -3$

8 Integrate using double angle formulae (All boards)

Use a double angle formula to find
$\int \cos^2 5x \, dx$.

$\cos 2\theta = 2\cos^2\theta - 1$, so $\cos^2 5x = \frac{1}{2}(1 + \cos 10x)$

Integrate term by term

Answer: $\frac{x}{2} + \frac{\sin 10x}{20} + c$

9 Equilibrium of forces given as vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-5\mathbf{i} + 2\mathbf{j})$ N, $(p\mathbf{i} + q\mathbf{j})$ N and $(\mathbf{i} + \mathbf{j})$ N act on a particle which is in equilibrium. $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors. Find the values of p and q.

Sum of forces = 0:
i components: $p + (-5) + (1) = 0$
j components: $q + (2) + (1) = 0$
Answer: $p = 4$, $q = -3$

10 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(40, p)$. H_0: $p = 0.5$, H_1: $p > 0.5$.
Find the critical region for a test at the 10% significance level.

$P(X \geq 25) = 0.0769 \leq 0.1$
$P(X \geq 24) = 0.134 > 0.1$
Answer: $X \geq 25$