Day 87 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\sqrt{2} \times \sqrt{28}$, giving your answer in the form $p\sqrt{q}$, or as an integer.

$\sqrt{2} \times \sqrt{28} = \sqrt{56}$

$= 2\sqrt{14}$

Answer: $2\sqrt{14}$

2 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of
(a) $\arctan(-1)$ (b) $\arcsin(1)$

(a) $y = \arctan(-1)$ means $\tan y = -1$ with $-\frac{\pi}{2} < y < \frac{\pi}{2}$: $y = -\frac{\pi}{4}$

(b) $y = \arcsin(1)$ means $\sin y = 1$ with $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$: $y = \frac{\pi}{2}$

Answer: (a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

3 Constant acceleration: $v^2 = u^2 + 2as$ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with constant acceleration. Its speed increases from 7 m s^{-1} to 16 m s^{-1} as it travels 41.4 m. Find the acceleration.

$v^2 = u^2 + 2as$

$16^2 = 7^2 + 2 \times a \times 41.4$

$207 = 82.8\,a$

Answer: 2.5 m s^{-2}

4 Solve trig equation in radians (calculator) (All boards)

Solve $3\cos x = -2$ for $0 \leq x \leq 2\pi$.
Give your answers to 3 significant figures.

$\cos x = -\frac{2}{3}$

Reference angle = $\cos^{-1}(0.6667) = 0.8411$ radians
cos is negative in the second and third quadrants.
$x = 2.30, 3.98$
Answer: x = 2.30, 3.98

5 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 8^2)$. H_0: $\mu = 140$, H_1: $\mu < 140$.

A sample of 36 has mean $\bar{x} = 136.13$.
Test at the 5% level. Critical value: $z = -1.645$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (136.13 - 140) \div (8 \div 6)$

$z = -2.90$

-2.90 is inside the critical region, so reject H_0

Answer: z = −2.90; reject H_0

6 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = -3x - 8$.
Find $f^{-1}(x)$.

$y = -3x - 8$, so $y + 8 = -3x$

$x = \frac{y + 8}{-3}$

Answer: $f^{-1}(x) = \frac{x + 8}{-3}$

7 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(20, 4^2)$. Given that $P(X < a) = 0.7$, find a to 3 s.f.

$P(Z < z) = 0.7$, so $z = 0.5244$ (from the inverse normal function)

$a = 20 + 0.5244 \times 4$

Answer: 22.1

8 Maximum value from R form (All boards)

$f(x) = 12 \sin x + 5 \cos x + 3$.

Write f(x) in the form $R \sin(x + \alpha) + 3$, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{12^2 + 5^2} = 13$, $\tan \alpha = \frac{5}{12}$ so $\alpha = 22.6°$

$f(x) = 13 \sin(x + 22.6°) + 3$

$\sin(x + \alpha)$ has maximum value 1, so f(max) = 13 + 3

Answer: f(x) = 13 sin(x + 22.6°) + 3; maximum 16

9 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A stone is projected from level ground with velocity $(8\mathbf{i} + 19.6\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.

(a) Find the time of flight.

(b) Find the horizontal range.

(a) Vertically: $0 = 19.6\,t - 4.9t^2$

$t = \frac{19.6}{4.9} = 4$

(b) Range $= 8 \times 4$

Answer: (a) 4 s (b) 32 m

10 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = -2$ and $y = -1$, so $a = -2$ and $c = -1$.

Through $(0, -3)$: $-3 = \frac{k}{0 - (-2)} + -1$

$\frac{k}{2} = -2$, so $k = -4$

Answer: a = −2, c = −1, k = −4