Day 86 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Natural logs and e^x (All boards)

Solve $e^{4x+1} = 5$.
Give your answer to 3 significant figures.

Take natural logs: $4x + 1 = \ln 5 = 1.609$
$x = 0.152$
Answer: x = 0.152

2 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show, by a counter-example, that the statement "If $x^2 > y^2$ then $x > y$" is false.

Try $x = -4$, $y = 1$.
$x^2 = 16$ and $y^2 = 1$, so $x^2 > y^2$.
But $x < y$, so the statement is false.
Answer: e.g. x = −4, y = 1

3 Large data set: handling readings (AQA, Edexcel, OCR, MEI, Eduqas)

Seven daily sunshine readings (hours) from the Large Data Set are: 5, 2.2, n/a, 0.2, n/a, 3.2, 0.6.
Here 'n/a' means no reading was recorded.
Leaving these out, find the mean to 3 s.f.

Leave out the 2 'n/a' readings, leaving 5 values
Total = 11.2
Mean = 11.2 ÷ 5
Answer: 2.24 hours

4 Exponential graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 4 \times 3^x - 36$.
Write down the equation of the asymptote and the coordinates where the curve meets the axes.

As $x \to -\infty$, $3^x \to 0$, so $y \to -36$: asymptote $y = -36$.
$x = 0$: $y = 4 \times 3^0 - 36 = -32$
$y = 0$: $4 \times 3^x = 36$, so $3^x = 9$ and $x = 2$
Answer: y = −36 ; (0, −32), (2, 0)

5 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 5 values: 9, 8, 5, 8, 19
Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 49$, $\Sigma x^2 = 595$, $n = 5$
Mean = 49 ÷ 5 = 9.800
Variance = $\frac{595}{5} - (9.800)^2 = 22.96$
Standard deviation = $\sqrt{22.96}$
Answer: mean = 9.80, standard deviation = 4.79

6 Stationary points from a sketch (diagram) (All boards)

The diagram shows the curve $y = x^3 - 3x + 2$.
A and B are turning points. Find the coordinates of A and B.

$dy/dx = 3x^2 - 3$
$dy/dx = 0$ gives $x = -1$ or $x = 1$
Substitute: $y = 4$ at $x = -1$ and $y = 0$ at $x = 1$
Answer: A (−1, 4), B (1, 0)

7 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that if $3n + 4$ is even, then n is even, where n is an integer.

Assume n is odd, so $n = 2k + 1$ for an integer k.
$3n + 4 = 6k + 3 + 4 = 2(3k + 3) + 1$
This is odd, which contradicts $3n + 4$ being even.
So the assumption is false: n must be even.
Answer: Proved

8 Work done and kinetic energy (CIE)

A constant force of 20 N pulls a crate 6 m along rough horizontal ground against a constant frictional force of 5 N. Find (a) the work done by the pulling force, (b) the work done against friction.

(a) Work done = force × distance = 20×6
(b) Work done against friction = 5×6
Answer: (a) 120 J (b) 30 J

9 Integrate using partial fractions (All boards)

$\frac{2x + 5}{(x + 4)(x + 3)} \equiv \frac{A}{x + 4} + \frac{B}{x + 3}$.

Find A and B, and hence find $\int \frac{2x + 5}{(x + 4)(x + 3)}\,dx$.

$2x + 5 \equiv A(x + 3) + B(x + 4)$
$x = -4$ gives $A = 3$; $x = -3$ gives $B = -1$
$\int \left(\frac{3}{x + 4} + \frac{-1}{x + 3}\right) dx = 3 \ln|x + 4| + -1 \ln|x + 3| + c$
Answer: $A = 3$, $B = -1$; $3 \ln|x + 4| - \ln|x + 3| + c$

10 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \le t \le 8$) is $v = (-3t^2 + 24t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 24 - 6t$

Maximum when $a = 0$: $t = \frac{24}{6} = 4$

$v = 24 \times 4 - 3 \times 4^2$
Answer: 48 m s^{-1} at $t = 4$