Day 85 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Solve basic trig equation (exact, degrees) (All boards)

Solve $\sin x = \frac{1}{2}$ for $0° \leq x \leq 180°$.

Reference angle: $\sin 30° = \frac{1}{2}$

sin is positive in the first and second quadrants.
Using the sin graph/CAST and the interval $0° \leq x \leq 180°$:
$x = 30°, 150°$
Answer: $x = 30°, 150°$

2 Histograms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t minutes, taken by some people to complete a puzzle.
Find the number of people who took between 5 and 20 minutes.

Frequency = frequency density × class width
$5 \leq t < 15$: $3 \times 10 = 30$
$15 \leq t < 20$: $3 \times 5 = 15$
Total = 30 + 15
Answer: 45

3 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Rationalise the denominator of $\frac{1}{4 + \sqrt{10}}$.

Write your answer in the form $p + q\sqrt{10}$.

Multiply top and bottom by $4 - \sqrt{10}$:
$\frac{1(4 - \sqrt{10})}{(4)^2 - (\sqrt{10})^2} = \frac{1(4 - \sqrt{10})}{6}$
Answer: $\frac{2}{3} - \frac{1}{6}\sqrt{10}$

4 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix}$.

Find the coordinates of the point on the line where t = 2.

$\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} + 2\begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix}$
$= \begin{pmatrix} 9 \\ 6 \\ -7 \end{pmatrix}$
Answer: (9, 6, −7)

5 Moments: rod on a pivot with a particle (AQA, Edexcel, OCR, MEI)

A uniform rod AB of mass 15 kg and length 2 m is supported by a pivot 0.75 m from A. A particle of mass m kg is attached at A and the rod rests horizontally in equilibrium. Find m.

Moments about the pivot:
$m\,g \times 0.75 = 15\,g \times (1 - 0.75)$
$m = \frac{15 \times 0.25}{0.75}$
Answer: 5 kg

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6 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form k(x − a)(x − b)(x − c).

The curve crosses the x-axis at x = 1, 3 and 4.
f(x) = k(x − 1)(x − 3)(x − 4)
At (0, −24): k × (−1) × (−3) × (−4) = −24
k = 2
Answer: f(x) = 2(x − 1)(x − 3)(x − 4)

7 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train starts from rest and accelerates uniformly at 2 m s^{-2} for 5 s, then travels at constant speed for a further 10 s. Find the total distance travelled.

Speed reached: v = 2 × 5 = 10 m s^{-1}
Stage 1: s = $\frac{1}{2}$ × 10 × 5 = 25
Stage 2: s = 10 × 10 = 100
Answer: 125 m

8 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = 1 + \frac{4}{x_n}$, with $x_0 = 2$.

Find x_1, x_2 and x_3 to 3 decimal places.

x_1 = 3.0000
x_2 = 2.3333
x_3 = 2.7143
Answer: x_1 = 3.000, x_2 = 2.333, x_3 = 2.714

9 Normal approximation to the binomial (CIE)

X ~ B(40, 0.6). Use a normal approximation to find P(X ≥ 24), to 3 s.f.

np = 24 and npq = 9.6, so use Y ~ N(24, 9.6)
Continuity correction: P(Y > 23.5)
z = (23.5 − 24) ÷ $\sqrt{9.6}$
Answer: 0.564

10 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(x − 2) is a factor of $x^3 + kx^2 − 3x − 18$.
Find the value of k.

Let $f(x) = x^3 + kx^2 − 3x − 18$. Then f(2) = 0:
8 + 4k − 24 = 0
4k = 16
Answer: k = 4