Day 84 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through A(0, −5) and B(−2, 5), in the form ax + by + c = 0, where a, b and c are integers.

Gradient = $\frac{5-(-5)}{-2-0} = -5$

$y - (-5) = -5(x - 0)$

Multiply out and rearrange to integer coefficients.

Answer: $5x + y + 5 = 0$

2 Constant acceleration: time to reach a speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line increases its speed from 2 m s^{-1} to 4 m s^{-1} with constant acceleration 1 m s^{-2}. Find the time taken.

$v = u + at$

$4 = 2 + 1\,t$

Answer: 2 s

3 Discrete random variable: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has $P(X = x) = k(x^2 + 2)$ for x = 0, 1, 2, 3, 4.

(a) Find the value of k.

(b) Find $P(X \geq 3)$.

(a) Probabilities sum to 1: $k(2 + 3 + 6 + 11 + 18) = 1$

$k = \frac{1}{40}$

(b) $P(X \geq 3) = \frac{11}{40} + \frac{18}{40}$

Answer: (a) $k = \frac{1}{40}$ (b) $\frac{29}{40}$

4 Area between a line and a curve (All boards)

The curve $y = x^2$ and the line y = 2x meet at the origin and at P.

Find the area enclosed between them.

$x^2 = 2x$ gives x = 0 or x = 2

Area = $\int_0^2 (2x - x^2)\,dx = [\frac{2x^2}{2} - \frac{x^3}{3}]$ from 0 to 2

$= \frac{2^3}{2} - \frac{2^3}{3} = \frac{4}{3}$

Answer: $\frac{4}{3}$

5 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x + 1 \leq -20$.

Solve $-x^2 + x + 20 = 0$: $-(x + 4)(x - 5) = 0$

Critical values x = −4 and x = 5.

The graph is an ∩-shaped parabola, so take the regions outside the roots.

Answer: $x \leq -4$ or $x \geq 5$

6 Solve trig equation in radians (calculator) (All boards)

Solve $5 \sin x = -2$ for $0 \leq x \leq 2\pi$.

Give your answers to 3 significant figures.

$\sin x = -\frac{2}{5}$

Reference angle = $\sin^{-1}(0.4000) = 0.4115$ radians

sin is negative in the third and fourth quadrants.

x = 3.55, 5.87

Answer: x = 3.55, 5.87

7 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 2x + k$ is a tangent to the curve
$y = x^2 + 3$.
Find the value of k.

$x^2 + 3 = 2x + k$
$x^2 - 2x + (3 - k) = 0$
A tangent touches once: discriminant = 0
$(-2)^2 - 4(3 - k) = 0$
Answer: k = 2

8 Vertical motion under gravity: thrown upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is thrown vertically upwards with speed 19.6 m s^{-1}. Take g = 9.8 m s^{-2}.
(a) Find the time taken to reach its greatest height.
(b) Find the greatest height reached.

(a) $v = u + at$: $0 = 19.6 - 9.8t$, so $t = 2$
(b) $v^2 = u^2 + 2as$: $0 = 19.6^2 - 2 \times 9.8 \times s$
$s = \frac{384.16}{19.6}$
Answer: (a) 2 s (b) 19.6 m

9 Venn diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities for events A and B.
Find the value of x.

Probabilities sum to 1
$x = 1 - (0.1 + 0.15 + 0.25)$
Answer: 0.5

10 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} + 2\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (1)(-3) + (3)(0) + (1)(2) = -1$
$|\mathbf{a}| = \sqrt{11}$, $|\mathbf{b}| = \sqrt{13}$
$\cos\theta = \frac{-1}{\sqrt{11} \times \sqrt{13}}$, so $\theta = 94.8°$
Answer: 94.8°