Day 83 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (0, −3) and B is (−6, −9).
Find (a) the midpoint of AB,
(b) the length of AB, as a surd in simplest form.

(a) Midpoint = $(\frac{0+(-6)}{2}, \frac{-3+(-9)}{2}) = (-3, -6)$

(b) $AB^2 = (-6)^2 + (-6)^2 = 36 + 36 = 72$
$AB = \sqrt{72} = 6\sqrt{2}$

Answer: (a) (−3, −6) (b) $6\sqrt{2}$

2 Velocity-time: constant deceleration to rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car decelerates uniformly from 24 m s^{-1} to rest in 8 s. Find (a) the deceleration, (b) the distance travelled while decelerating.

(a) $a = \frac{v-u}{t} = \frac{0-24}{8}$

(b) $s = \frac{(u+v)}{2} t = 12 \times 8$

Answer: (a) 3 m s^{-2} (b) 96 m

3 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence begins 2, −4, 8, ...
Find the common ratio and the 5th term.

$r = \frac{-4}{2} = -2$

$u_n = ar^{n-1}$, so $u_5 = 2 \times (-2)^4$
$= 32$

Answer: r = −2, u_5 = 32

4 Tree diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows probabilities for events R and L.
Find the probability that exactly one of R and L happens.

Exactly one: (R and L′) or (R′ and L)
$= 0.8 \times 0.9 + 0.2 \times 0.8$

Answer: 0.88

5 Solve trig equation in radians (calculator) (All boards)

Solve $3 \sin x = -1$ for $0 \leq x \leq 2\pi$.
Give your answers to 3 significant figures.

$\sin x = -\frac{1}{3}$

Reference angle = $\sin^{-1}(0.3333) = 0.3398$ radians
sin is negative in the third and fourth quadrants.
x = 3.48, 5.94

Answer: x = 3.48, 5.94

6 Conditional probability formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P(B) = 0.4 and P(A | B) = 0.1.
Find $P(A \cap B)$.

$P(A \cap B) = P(A \mid B) \times P(B)$
$= 0.1 \times 0.4$

Answer: 0.04

7 Horizontal projection from a cliff (AQA, Edexcel, OCR, MEI)

A ball is thrown horizontally at 19 m s^{-1} from the top of a vertical cliff of height 78.4 m above the sea. Take g = 9.8 m s^{-2}. Find (a) the time taken to reach the sea, (b) the horizontal distance of the landing point from the cliff.

(a) Vertically: u = 0, s = 78.4, a = 9.8

$78.4 = \frac{1}{2} \times 9.8 \times t^2$, so $t^2 = 16$

(b) Horizontal distance = speed × time = 19 × 4

Answer: (a) 4 s (b) 76 m

8 Quadratic in sin or cos (All boards)

Solve $\cos^2 x - \cos x = 0$ for $0° \leq x \leq 360°$.

Factorise: $\cos x(\cos x - 1) = 0$ (as a quadratic in cos x)

$\cos x = 0$ or $\cos x = 1$

$x = 0°, 90°, 270°, 360°$

Answer: x = 0°, 90°, 270°, 360°

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = x + k$ is a tangent to the curve $y = x^2 - x - 3$.

Find the value of k.

$x^2 - x - 3 = x + k$

$x^2 - 2x + (-3 - k) = 0$

A tangent touches once: discriminant = 0

$(-2)^2 - 4(-3 - k) = 0$

Answer: k = −4

10 Solve with a shifted angle (All boards)

Solve $\sin(x + 45°) = \frac{\sqrt{2}}{2}$ for $0° \leq x \leq 360°$.

Let $u = x + 45°$, so $45° \leq u \leq 405°$

$\sin u = \frac{\sqrt{2}}{2}$: $u = 45°, 135°, 405°$

$x = u - 45°$: $x = 0°, 90°, 360°$

Answer: x = 0°, 90°, 360°