Day 82 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Describing transformations of graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of y = f(4x).

y = f(4x) is obtained from y = f(x) by a:

stretch parallel to the x-axis, scale factor $\frac{1}{4}$

Answer: stretch parallel to the x-axis, scale factor $\frac{1}{4}$

2 Natural logs and e^x (All boards)

Solve $e^{4x+2} = 12$.

Give your answer to 3 significant figures.

Take natural logs: 4x + 2 = ln 12 = 2.485

x = 0.121

Answer: x = 0.121

3 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (−4, −1) and B is (0, 5). AB is a diameter of a circle.

Find the equation of the circle.

Centre = midpoint of AB = (−2, 2)

$r^2 = (0 - (-2))^2 + (5 - 2)^2 = 13$

Equation: $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x + 2)^2 + (y - 2)^2 = 13$

4 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve y = ln x + 1 from x = 1 to x = 5 divided into 4 strips of equal width.

Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

h = 1; y values: 1.000, 1.693, 2.099, 2.386, 2.609

Area ≈ $\frac{1}{2}$[1.000 + 2.609 + 2(6.178)]

= 7.98

Answer: 7.98

5 Moments: rod on a pivot with a particle (AQA, Edexcel, OCR, MEI)

A uniform rod AB of mass 15 kg and length 4 m is supported by a pivot 0.25 m from A. A particle of mass m kg is attached at A and the rod rests horizontally in equilibrium. Find m.

Moments about the pivot:

m g × 0.25 = 15 g × (2 − 0.25)

$m = \frac{15 \times 1.75}{0.25}$

Answer: 105 kg

6 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 2^2)$. H_0: $\mu = 90$, H_1: $\mu < 90$.

A sample of 9 has mean $\bar{x} = 87.8$.

Test at the 5% level. Critical value: z = −1.645.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (87.8 - 90) \div (2 \div 3)$

z = −3.30

−3.30 is inside the critical region, so reject H_0

Answer: z = −3.30; reject H_0

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7 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + \frac{1}{2}x)^{-1}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -1$, $y = \frac{1}{2}x$

$= 1 + (-1)(\frac{1}{2}x) + \frac{(-1)((-2))}{2}(\frac{1}{2}x)^2$

$= 1 - \frac{1}{2}x + \frac{1}{4}x^2$

Valid when $|\frac{1}{2}x| < 1$, i.e. $|x| < 2$

Answer: $1 - \frac{1}{2}x + \frac{1}{4}x^2$; $|x| < 2$

8 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(10, p). H_0: p = 0.2, H_1: p > 0.2.

Find the critical region for a test at the 10% significance level.

$P(X \geq 5) = 0.0328 \leq 0.1$

$P(X \geq 4) = 0.121 > 0.1$

Answer: $X \geq 5$

9 Velocity-time graph: acceleration and distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a car over 21 seconds as it moves in a straight line.

(a) Find its acceleration in the first 4 s.

(b) Find the total distance travelled.

(a) acceleration = gradient = $\frac{10}{4}$

(b) distance = area of trapezium

$= \frac{1}{2}(12 + 21) \times 10$

Answer: (a) 2.5 m s^{-2} (b) 165 m

10 Identity then solve ($\sin^2$ to cos) (All boards)

Solve $-\cos^2 x + \sin x + 1 = 0$

for $0° \leq x \leq 360°$.

Use $\cos^2 x = 1 - \sin^2 x$: $\sin^2 x + \sin x = 0$

Factorise and solve: $\sin x = -1$ or $\sin x = 0$

$x = 0°, 180°, 270°, 360°$

Answer: $x = 0°, 180°, 270°, 360°$