Day 81 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Pythagorean identity: find cos and tan (All boards)

θ is acute and $\sin\theta = \frac{3}{5}$.

Find the exact values of $\cos\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$, so $\cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$

θ is acute, so $\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4}$

Answer: $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$

2 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle starts from rest and moves in a straight line with constant acceleration 1 m s^{-2}.
It travels 8 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with $u = 0$

$8 = \frac{1}{2} \times 1 \times t^2$

$t^2 = 16$

Answer: 4 s

3 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (3, −3) and B is (3, −9).
Find (a) the midpoint of AB,
(b) the length of AB, as a surd in simplest form.

(a) Midpoint $= \left(\frac{3+3}{2}, \frac{-3+(-9)}{2}\right) = (3, -6)$

(b) $AB^2 = (0)^2 + (-6)^2 = 0 + 36 = 36$

$AB = \sqrt{36} = 6$

Answer: (a) (3, −6) (b) 6

4 Frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a histogram, the class 0 to 20 has frequency 10 and is drawn with a bar of height 2.5 cm.
The class 20 to 30 has frequency 10. Find the height of its bar.

Frequency density = frequency ÷ class width
First bar: 10 ÷ 20 = 0.5, so 2.5 cm represents 0.5
Second bar: 10 ÷ 10 = 1

Height $= 2.5 \times \frac{1}{0.5}$

Answer: 5 cm

5 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x + 2)^2 + (y + 4)^2 = 36$.
Determine whether the point P(8, −3) is inside, outside or on the circle.

Distance^2 from the centre (−2, −4) to P:
$(8 - (-2))^2 + (-3 - (-4))^2 = 100 + 1 = 101$
$r^2 = 36$, and $101 > 36$.

Answer: P is outside the circle

Day 81 **A Level Maths: C Tier 10-a-day: Model Solutions**

6 Equation of a tangent or normal (All boards)

Find the equation of the normal to the curve
$y = e^{2x} + x$ at the point where $x = 0$.

$dy/dx = 2\exp(2x) + 1$, so the gradient of the curve at $x = 0$ is 3
At $x = 0$, $y = 1$

Gradient of normal $= -\frac{1}{3}$

$y - 1 = (-\frac{1}{3})(x - 0)$

Answer: $y = -\frac{1}{3}x + 1$

7 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \leq t \leq 6$) is $v = (-t^2 + 6t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 6 - 2t$

Maximum when $a = 0$: $t = \frac{6}{2} = 3$

$v = 6 \times 3 - 1 \times 3^2$

Answer: 9 m s^{-1} at t = 3

8 Quotient rule (All boards)

Use the quotient rule to find the derivative of
$y = \frac{e^x}{3x + 1}$.

Quotient rule: if $y = \frac{u}{v}$ then $dy/dx = \frac{u'v - uv'}{v^2}$

Differentiate the numerator and denominator, then substitute and simplify.

Answer: $dy/dx = \frac{e^x(3x + -2)}{(3x + 1)^2}$

9 Discrete random variable: E(X) and Var(X) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has this distribution.
x: 0, 3, 7, 8
P(X = x): 0.5, 0.2, 0.1, 0.2
Find E(X) and Var(X).

$E(X) = \Sigma xp = 0\times0.5 + 3\times0.2 + 7\times0.1 + 8\times0.2 = 2.9$
$E(X^2) = \Sigma x^2 p = 19.5$
$Var(X) = 19.5 - 2.9^2$
Answer: E(X) = 2.9, Var(X) = 11.09

10 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both
$2x + 1 < -9$ and $x^2 + 8x + 12 < 0$.

$2x + 1 < -9$ gives $x < -5$.
$x^2 + 8x + 12 < 0$: $(x + 6)(x + 2) < 0$, so $-6 < x < -2$.
Both are true where the two ranges overlap.
Answer: $-6 < x < -5$