Day 80 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Mutually exclusive events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B are mutually exclusive, with P(A) = 0.1 and P(B) = 0.45.
(a) Find P(A ∪ B).
(b) Find the probability that neither A nor B happens.

(a) Mutually exclusive, so P(A ∩ B) = 0
P(A ∪ B) = 0.1 + 0.45 = 0.55
(b) 1 − 0.55 = 0.45
Answer: (a) 0.55 (b) 0.45

2 Amplitude and period from a graph (diagram) (All boards)

The diagram shows the graph of $y = a \sin bx$ for $0° \leq x \leq 360°$, where a and b are positive integers.
Find a and b.

The maximum value of y is 5, so a = 5

The graph completes 2 full cycles in 360°, so the period is 180° and $b = \frac{360}{180} = 2$

Answer: a = 5, b = 2

3 Free fall: dropped from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is dropped from rest from the top of a tower of height 78.4 m. Take g = 9.8 m s^{-2} and ignore air resistance. Find (a) the time taken to reach the ground, (b) its speed on hitting the ground.

(a) $s = ut + \frac{1}{2}at^2$: $78.4 = \frac{1}{2} \times 9.8 \times t^2$

$78.4 = 4.9t^2$, so $t^2 = 16$
(b) $v = u + at = 9.8 \times 4$
Answer: (a) 4 s (b) 39.2 m s^{-1}

4 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence has first term 5 and common ratio 3. Find the first term that exceeds 10000.

$u_n = 5 \times 3^{n-1} > 10000$
$u_7 = 3645$ and $u_8 = 10935$
Answer: $u_8 = 10935$

5 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample of 15 patients is taken from a numbered list of 225.
The first one selected is number 15. Find the number of the 5th one selected.

Interval = 225 ÷ 15 = 15
Number = 15 + 4 × 15
Answer: 75

6 Partial fractions: find the constants (All boards)

Express $\frac{-4x - 13}{(x + 4)(x + 1)}$ in the form $\frac{A}{x + 4} + \frac{B}{x + 1}$.
Find the values of A and B.

$-4x - 13 \equiv A(x + 1) + B(x + 4)$
$x = -4$: $3 = A(-3)$, so $A = -1$
$x = -1$: $-9 = B(3)$, so $B = -3$
Answer: A = −1, B = −3

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7 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic series has first term 9 and common difference 2. Find the smallest n for which the sum of the first n terms exceeds 800.

$S_n = \frac{n}{2}[18 + 2(n - 1)] = \frac{n(2n + 16)}{2}$

$S_{24} = 768$ and $S_{25} = 825$

The sum first exceeds 800 when n = 25.

Answer: n = 25

8 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \leq t \leq 6$) is $v = (-3t^2 + 18t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 18 - 6t$

Maximum when a = 0: $t = \frac{18}{6} = 3$

$v = 18 \times 3 - 3 \times 3^2$

Answer: 27 m s^{-1} at t = 3

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that 15x + 5y = 104.

Assume that integers x and y exist with 15x + 5y = 104.

Then 5(3x + y) = 104, so 104 would be a multiple of 5.

But $104 = 5 \times 20 + 4$, so it is not a multiple of 5.

This is a contradiction, so no such integers exist.

Answer: Proved

10 Integrate using partial fractions (All boards)

$\frac{-x - 5}{(x + 4)(x + 3)} \equiv \frac{A}{x + 4} + \frac{B}{x + 3}$.

Find A and B, and hence find $\int \frac{-x - 5}{(x + 4)(x + 3)}\,dx$.

$-x - 5 \equiv A(x + 3) + B(x + 4)$

$x = -4$ gives A = 1; $x = -3$ gives B = −2

$\int \left(\frac{1}{x + 4} + \frac{-2}{x + 3}\right) dx = 1 \ln|x + 4| + -2 \ln|x + 3| + c$

Answer: A = 1, B = −2; $\ln|x + 4| - 2 \ln|x + 3| + c$