Day 8 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = -5\mathbf{i} - 12\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{5^2 + 12^2} = 13$

(b) Acute angle = $\tan^{-1}(\frac{12}{5}) = 67.4°$

p is in the third quadrant, so the angle is 247.4°

Answer: (a) 13 (b) 247.4°

2 Resultant of perpendicular forces (All boards)

Two forces of magnitudes 6 N and 8 N act on a boat at right angles to each other. Find (a) the magnitude of the resultant, (b) the angle it makes with the 6 N force, to 1 d.p.

(a) $R^2 = 6^2 + 8^2 = 100$

$R = \sqrt{100}$

(b) $\tan\theta = \frac{8}{6}$

Answer: (a) 10 N (b) 53.1°

3 Solving quadratics: surd answers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - 10x - 5 = 0$, giving your answers in the form $p \pm \sqrt{q}$.

Complete the square: $(x - 5)^2 - 25 - 5 = 0$

$(x - 5)^2 = 30$

$x - 5 = \pm\sqrt{30}$

Answer: $x = 5 \pm \sqrt{30}$

4 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(50, p) and E(X) = 5.
Find p and Var(X).

np = 5, so p = 5 ÷ 50 = 0.1

Var(X) = npq = 5 × 0.9

Answer: p = 0.1, Var(X) = 4.5

5 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = 1 + \frac{4}{x_n}$, with $x_0 = 1$.

Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 5.0000$

$x_2 = 1.8000$

$x_3 = 3.2222$

Answer: $x_1 = 5.000$, $x_2 = 1.800$, $x_3 = 3.222$

6 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(4, −1) lies on the curve y = f(x).
Write down the coordinates of the image of P on
(a) y = f(2x) (b) y = f(x − 3)

(a) y = f(2x) divides x by 2, so P maps to (2, −1).
(b) y = f(x − 3) adds 3 to x, so P maps to (7, −1).

Answer: (a) (2, −1) (b) (7, −1)

7 Identity then solve (sin² to cos) (All boards)

Solve $-2\cos^2 x - 3\sin x + 3 = 0$
for $0° \leq x \leq 360°$.

Use $\cos^2 x = 1 - \sin^2 x$: $2\sin^2 x - 3\sin x + 1 = 0$

Factorise and solve: $\sin x = \frac{1}{2}$ or $\sin x = 1$

$x = 30°, 90°, 150°$

Answer: x = 30°, 90°, 150°

8 Moments: balancing a plank (AQA, Edexcel, OCR, MEI)

A light plank rests on a pivot. A child of mass 30 kg sits 2 m from the pivot on one side. Where must a child of mass 25 kg sit on the other side for the plank to balance horizontally?

Moments about the pivot:
$30\text{ g} \times 2 = 25\text{ g} \times d$
$d = \frac{30 \times 2}{25}$

Answer: 2.4 m from the pivot

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x + k$ is a tangent to the curve
$y = x^2 - 5x - 1$.
Find the value of k.

$x^2 - 5x - 1 = -x + k$
$x^2 - 4x + (-1 - k) = 0$
A tangent touches once: discriminant = 0
$(-4)^2 - 4(-1 - k) = 0$

Answer: k = −5

10 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: $p = 0.2$, H_1: $p \neq 0.2$. In a sample, $x = 6$.
Test at the 10% significance level.

Two-tailed test, so compare with $0.1 \div 2 = 0.05$
$P(X \geq 6) = 0.196$
$0.196 > 0.05$, so do not reject H_0

Answer: Do not reject H_0 ($P(X \geq 6) = 0.196$)