Day 79 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Driving force and resistance (All boards)

A car of mass 1000 kg is driven along a straight horizontal road by a driving force of 600 N against a constant resistance of 400 N.

Find the acceleration of the car.

Resultant force = 600 − 400 = 200 N

F = ma: 200 = 1000 a

Answer: 0.2 m s^{-2}

2 Maximum value from R form (All boards)

f(x) = 9 sin x + 12 cos x + 6.

Write f(x) in the form R sin(x + α) + 6, with α in degrees to 1 d.p., and state the maximum value of f(x).

R = $\sqrt{9^2 + 12^2}$ = 15, tan α = $\frac{12}{9}$ so α = 53.1°

f(x) = 15 sin(x + 53.1°) + 6

sin(x + α) has maximum value 1, so f(max) = 15 + 6

Answer: f(x) = 15 sin(x + 53.1°) + 6; maximum 21

3 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 3)^2 + (y + 4)^2 = 8$.

Write down the centre and the radius of the circle.

Compare with $(x - a)^2 + (y - b)^2 = r^2$.

Centre (3, −4); $r^2 = 8$ so $r = 2\sqrt{2}$

Answer: Centre (3, −4), radius $2\sqrt{2}$

4 Integrate using partial fractions (All boards)

$\frac{3x + 9}{(x + 4)(x + 1)} \equiv \frac{A}{x + 4} + \frac{B}{x + 1}$.

Find A and B, and hence find $\int \frac{3x + 9}{(x + 4)(x + 1)}\,dx$.

3x + 9 ≡ A(x + 1) + B(x + 4)

x = −4 gives A = 1; x = −1 gives B = 2

$\int \left(\frac{1}{x + 4} + \frac{2}{x + 1}\right) dx = 1 \ln|x + 4| + 2 \ln|x + 1| + c$

Answer: A = 1, B = 2; ln |x + 4| + 2 ln |x + 1| + c

5 Standard deviation from summary data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 12 values, $\Sigma x = 144$ and $\Sigma x^2 = 2040$.

Find the mean and the standard deviation.

Mean = 144 ÷ 12 = 12

Variance = $\frac{2040}{12} - 12^2 = 170 - 144 = 26$

Standard deviation = $\sqrt{26}$

Answer: mean = 12, standard deviation = 5.10

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6 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - 2x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -\frac{1}{2}$, $y = -2x$

$= 1 + (-\frac{1}{2})(-2x) + \frac{(-\frac{1}{2})((-\frac{3}{2}))}{2}(-2x)^2$

$= 1 + x + \frac{3}{2}x^2$

Valid when $|-2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 + x + \frac{3}{2}x^2$; $|x| < \frac{1}{2}$

7 Resultant of perpendicular forces (All boards)

Two forces of magnitudes 8 N and 15 N act on a boat at right angles to each other. Find (a) the magnitude of the resultant, (b) the angle it makes with the 8 N force, to 1 d.p.

(a) $R^2 = 8^2 + 15^2 = 289$

$R = \sqrt{289}$

(b) $\tan\theta = \frac{15}{8}$

Answer: (a) 17 N (b) 61.9°

8 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 4) and (4, 324).
Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 4$

$x = 4$: $4 \times b^4 = 324$, so $b^4 = 81$

$b = 3$

Answer: a = 4, b = 3

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -3x + k$ is a tangent to the curve
$y = x^2 + 3x + 6$.
Find the value of k.

$x^2 + 3x + 6 = -3x + k$

$x^2 + 6x + (6 - k) = 0$

A tangent touches once: discriminant = 0

$(6)^2 - 4(6 - k) = 0$

Answer: k = −3

10 Coding (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The values x are coded using $y = (x - 10) \div 2$.

The coded values have mean $\bar{y} = 0.9$ and standard deviation 1.1.
Find the mean and standard deviation of x.

$x = 10 + 2y$

Mean: $10 + 2 \times 0.9 = 11.8$

Standard deviation: $2 \times 1.1 = 2.2$ (adding 10 has no effect)

Answer: mean = 11.8, standard deviation = 2.2