Day 78 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 2)^2 + (y - 1)^2 = 4$.
Write down the centre and the radius of the circle.

Compare with $(x - a)^2 + (y - b)^2 = r^2$.
Centre (2, 1); $r^2 = 4$ so $r = 2$
Answer: Centre (2, 1), radius 2

2 Mean from a frequency table (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises the number of goals scored in a game by 50 people.
Goals, x: 1, 2, 3, 4, 5
Frequency, f: 21, 4, 6, 8, 11
Find the mean.

$\Sigma fx = 21\times1 + 4\times2 + 6\times3 + 8\times4 + 11\times5 = 134$
Mean = $\Sigma fx \div \Sigma f = 134 \div 50$
Answer: 2.68

3 Particle on a smooth table connected to a hanging particle (All boards)

A 8 kg particle on a smooth horizontal table is joined by a light string over a smooth pulley to a hanging 2 kg particle, as shown. The system is released from rest. Take $g = 9.8$ m s^{-2}. Find the tension.

Table particle: $T = 8a$
Hanging particle: $2 \times 9.8 - T = 2a$
Adding: $19.6 = 10a$
Answer: 15.68 N

4 Solve a logarithmic equation (two logs) (All boards)

Solve $\ln(x + 6) - \ln x = \ln 4$.

$\ln \frac{x+6}{x} = \ln 4$, so $\frac{x+6}{x} = 4$
$x + 6 = 4x$, so $6 = 3x$
$x = 2$
Answer: $x = 2$

5 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(-5\mathbf{j})$ N and $(12\mathbf{i} + 9\mathbf{j})$ N act on a particle of mass 4 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(-5\mathbf{j}) + (12\mathbf{i} + 9\mathbf{j})$
= $(12\mathbf{i} + 4\mathbf{j})$ N
F = ma: $a = \frac{1}{4}(12\mathbf{i} + 4\mathbf{j})$
Answer: $(3\mathbf{i} + \mathbf{j})$ m s^{-2}

6 Equation of a tangent or normal (All boards)

Find the equation of the normal to the curve
$y = 3x^3 - 4x^2 - 5$ at the point where $x = 3$.

dy/dx = 9x^2 − 8x, so the gradient of the curve at x = 3 is 57
At x = 3, y = 40
Gradient of normal = $-\frac{1}{57}$
$y - 40 = (-\frac{1}{57})(x - 3)$
Answer: $y = -\frac{1}{57}x + \frac{761}{19}$

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7 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: $p = 0.6$, H_1: $p < 0.6$.

Find the critical region for a test at the 5% significance level.

$P(X \leq 7) = 0.0210 \leq 0.05$

$P(X \leq 8) = 0.0565 > 0.05$

Answer: $X \leq 7$

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -4x + k$ is a tangent to the curve $y = x^2 - 8x - 3$.

Find the value of k.

$x^2 - 8x - 3 = -4x + k$

$x^2 - 4x + (-3 - k) = 0$

A tangent touches once: discriminant = 0

$(-4)^2 - 4(-3 - k) = 0$

Answer: $k = -7$

9 Integration by substitution (All boards)

Use the substitution $u = x^2 + 4$ to find

$\int \frac{2x}{x^2 + 4}\, dx$.

$u = x^2 + 4$, $du = 2x\, dx$, so $x\, dx = \frac{1}{2}\, du$

$\int \frac{2}{2u}\, du = \frac{2}{2} \ln |u|$

Answer: $1 \ln (x^2 + 4) + c$

10 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations $x = 4t - 5$, $y = t^2 + 1$.

The point P has y-coordinate 5 and positive t.

Find the coordinates of P.

$t^2 + 1 = 5$, so $t^2 = 4$, giving $t = 2$ ($t > 0$)

$x = 4 \times 2 - 5 = 3$

Answer: (3, 5)