Day 76 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Large data set: handling readings (AQA, Edexcel, OCR, MEI, Eduqas)

Seven daily sunshine readings (hours) from the Large Data Set are: n/a, 2.6, 0.6, 0.4, n/a, 0, 1.8.
Here 'n/a' means no reading was recorded.
Leaving these out, find the mean to 3 s.f.

Leave out the 2 'n/a' readings, leaving 5 values
Total = 5.4
Mean = 5.4 ÷ 5
Answer: 1.08 hours

2 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = 2x^3 - 10x^2 + 9x + 6$ by $(x - 3)$.
Find the quotient and the remainder.

Use synthetic division with a = 3:
Bottom row: 2, −4, −3, −3
Quotient: $2x^2 - 4x - 3$
Remainder: −3
Answer: Quotient $2x^2 - 4x - 3$, remainder −3

3 Work done and kinetic energy (CIE)

A particle of mass 2 kg changes its speed from 1 m s^{-1} to 7 m s^{-1}. Find the increase in its kinetic energy.

$KE = \frac{1}{2}mv^2$

Increase = $\frac{1}{2} \times 2 \times 7^2 - \frac{1}{2} \times 2 \times 1^2$

Answer: 48 J

4 Double angle equation (All boards)

Solve $\cos 2x = \frac{\sqrt{3}}{2}$ for $0° \leq x \leq 180°$.

Let u = 2x, so $0° \leq u \leq 360°$

$\cos u = \frac{\sqrt{3}}{2}$: u = 30°, 330°, 390°, 690°

Divide by 2: x = 15°, 165°
Answer: x = 15°, 165°

5 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 20 kg and length 6 m rests horizontally on supports at A and B. A load of mass 5 kg is placed on the beam 1.5 m from A. Take g = 9.8 m s^{-2}. Find the reaction at B.

Moments about A:
$R_B \times 6 = 20 \times 9.8 \times 3 + 5 \times 9.8 \times 1.5$
$R_B \times 6 = 661.5$
Answer: 110.25 N

6 Sector: arc length and area (diagram) (All boards)

The diagram shows sector OAB of a circle.
Find, in terms of π, (a) the arc length AB, (b) the area of the sector.

(a) Arc length = $r\theta = 8 \times \frac{3\pi}{4} = 6\pi$ cm

(b) Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times \frac{3\pi}{4} = 24\pi$ cm^2

Answer: (a) 6π cm (b) 24π cm^2

7 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 2x − 5 and g(x) = 2x + 3.
Find the value of gf(1).

f(1) = −3
g(−3) = −3
Answer: −3

8 Identity then solve (sin² to cos) (All boards)

Solve $-2\cos^2 x + \sin x + 1 = 0$
for $0° \leq x \leq 360°$.

Use $\cos^2 x = 1 - \sin^2 x$: $2\sin^2 x + \sin x - 1 = 0$

Factorise and solve: $\sin x = -1$ or $\sin x = \frac{1}{2}$

x = 30°, 150°, 270°
Answer: x = 30°, 150°, 270°

9 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + x^2 - 22x - 40$
(a) Show that (x − 5) is a factor of f(x).
(b) Factorise f(x) completely.

f(5) = 125 + 25 + (−110) + (−40) = 0, so (x − 5) is a factor.

$f(x) = (x - 5)(x^2 + 6x + 8)$

$x^2 + 6x + 8 = (x + 4)(x + 2)$

Answer: (b) f(x) = (x + 4)(x + 2)(x − 5)

10 Binomial: modelling conditions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is the number of heads when a fair coin is tossed 6 times.
Can X be modelled by a binomial distribution? Give a reason.

A binomial model needs: a fixed number of trials, two outcomes, constant probability, independent trials.
Here a fixed number of independent trials, each with probability 0.5 of a head.
Answer: Yes, a fixed number of independent trials, each with probability 0.5 of a head