Day 75 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(3, −2) lies on the curve y = f(x).
Write down the coordinates of the image of P on
(a) y = f(x − 4) (b) y = f(x + 4)

(a) y = f(x − 4) adds 4 to x, so P maps to (7, −2).
(b) y = f(x + 4) subtracts 4 from x, so P maps to (−1, −2).
Answer: (a) (7, −2) (b) (−1, −2)

2 Double angle: find sin 2A and cos 2A (All boards)

A is acute and sin A = $\frac{3}{5}$.

Find the exact values of sin 2A and cos 2A.

$\cos A = \sqrt{1 - \sin^2 A} = \frac{4}{5}$

$\sin 2A = 2 \sin A \cos A = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Answer: sin 2A = $\frac{24}{25}$, cos 2A = $\frac{7}{25}$

3 Differentiating velocity to find acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is v m s^{-1}, where $v = t^2 - 4t + 1$.
(a) Find its acceleration when t = 1.
(b) Find the time when its acceleration is zero.

$a = \frac{dv}{dt} = 2t - 4$

(a) t = 1: $a = 2 \times 1 - 4$

(b) $2t - 4 = 0$, so $t = \frac{4}{2}$

Answer: (a) −2 m s^{-2} (b) 2 s

4 Solve trig equation in radians (calculator) (All boards)

Solve 4 cos x = 3 for $0 \leq x \leq 2\pi$.
Give your answers to 3 significant figures.

$\cos x = \frac{3}{4}$

Reference angle = $\cos^{-1}(0.7500) = 0.7227$ radians
cos is positive in the first and fourth quadrants.
x = 0.723, 5.56
Answer: x = 0.723, 5.56

5 Discrete random variable: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has P(X = x) = k(6 − x) for x = 1, 2, 3, 4.
(a) Find the value of k.
(b) Find P(X ≥ 3).

(a) Probabilities sum to 1: k(5 + 4 + 3 + 2) = 1

$k = \frac{1}{14}$

(b) $P(X \geq 3) = \frac{3}{14} + \frac{2}{14}$

Answer: (a) k = $\frac{1}{14}$ (b) $\frac{5}{14}$

6 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form k(x − a)(x − b)(x − c).

The curve crosses the x-axis at x = −3, −2 and 1.
f(x) = k(x + 3)(x + 2)(x − 1)
At (0, 12): k × 3 × 2 × (−1) = 12
k = −2
Answer: f(x) = −2(x + 3)(x + 2)(x − 1)

7 Car towing a caravan (All boards)

A car of mass 1000 kg tows a caravan of mass 500 kg on a level road with acceleration 0.25 m s^{-2}. Resistances: 200 N on the car, 200 N on the caravan. Find the driving force and the tow bar tension.

Whole system: D − 200 − 200 = 1500 × 0.25
Caravan: T − 200 = 500 × 0.25
Answer: D = 775 N, T = 325 N

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = 3x + k is a tangent to the curve
$y = x^2 - x + 2$.
Find the value of k.

$x^2 - x + 2 = 3x + k$
$x^2 - 4x + (2 - k) = 0$
A tangent touches once: discriminant = 0
$(-4)^2 - 4(2 - k) = 0$
Answer: k = −2

9 Identity then solve ($\sin^2$ to cos) (All boards)

Solve $-4\sin^2 x + 3 = 0$
for $0° \leq x \leq 360°$.

Use $\sin^2 x = 1 - \cos^2 x$: $4\cos^2 x - 1 = 0$
Factorise and solve: $\cos x = -\frac{1}{2}$ or $\cos x = \frac{1}{2}$
x = 60°, 120°, 240°, 300°
Answer: x = 60°, 120°, 240°, 300°

10 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of size 14 is taken from 56 students: 40 Biology, 16 Chemistry.
How many Chemistry should be in the sample?

Number = $14 \times \frac{16}{56}$
= 4
Answer: 4